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Various mechanisms of CP violation in gauge theory are reviewed. We discuss the impact of recent developments associated with electric dipole moment(EDM) of neutron($D_n$), EDM of quarks($D_q$), chromo-EDM of quarks($D_q^c$), chromo-EDM of gluon($D_G^c$), EDM of electron($D_e$), and EDM of W boson($D_W$).

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# Part 0. Introduction

Soon after parity violation was proposed in 1956, Landau[1] introduced the combined CP invariance in 1957. He also observed that "a consequence of invariance with respect to combined inversion is that the weak interaction operators in the Lagrangian contain real coefficients". In 1964, Chirstenson, Cronin, Fitch and Turlay[2] observed the CP- violating $2\pi$ decay of the $K_L^0$ meson. Ever since then, particle theorists have been running around peering into every likely corner of their Lagrangian looking for elusive complex phases. At the same time, experimentalists are tuning up their instruments looking for any other appearance of CP violation. However, 26 years have passed, and the evidence for CP violation remains confined to the neutral kaon system while all the CP phenomenology can be very simply explained by a naive phenomenological model, called the superweak model, introduced by Wolfenstein[3] the same year CP violation was first observed.

While CP violation remained mysterious, the field of particle physics discovered its standard model. The standard model is a field theory basically dictated by gauge principle. The standard model based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ needed only two generations when it was first conceived. Kobayashi and Maskawa[4] realized that with only two generations there will be no room to put in a physically meaningful complex phase needed for CP violation. They seeked to generalize the theory to incorporate that and introduced the third generation. They showed that the theory with three generations has exactly one such CP violating phase. As the bottom quark and $\tau$ lepton were subsequently discovered this model of CP violation became the standard model of CP violation.

There were other attempts at generalizing the standard model to incorporate CP violation. In 1974, Lee[5] provided another way of looking at CP violation. He showed how CP can be broken spontaneously through the Higgs mechanism just like the gauge symmetry. He demonstrated the mechanism in a model with minimal extension of the Higgs sector of the standard model. In the model a second doublet of Higgs bosons is introduced in addition to the one in the standard model. In this minimal extension, the CP violation is mediated by the flavor changing neutral Higgs bosons. The experimental constraints require these bosons to be very heavy. As a result, the model behaves like a superweak model. Weinberg[6] suggested to eliminate the flavor changing neutral current by imposing a discrete symmetry in order to acheive so called "natural flavor conservation"[7]. In that case CP can be broken in a two generation model only when there are three doublets of Higgs bosons. The dominant CP-violating mechanism is mediated by the charged Higgs bosons. As a result, the model is milliweak in character. CP can be broken spontaneously or explicitly in the model [8, 9, 10, 11, 12]. For the case of spontaneous breaking, it is easy to show that there is only one CP violating phase in the charged Higgs sector and the K-M phase vanishes at the tree level even when there are three generations of fermions[10]. Being milliweak in character, the model predicts large CP violation in many experiments other than the neutral kaon system. In fact the model is under experimental assult on many fronts. For example the model predicts relatively large values for $\frac{\epsilon'}{\epsilon}$,[13] for CP violating correlation in semileptonic $K^0_{\mu 3}$ decay[14] and for the neutron electric dipole moment, $D_N$[15]. It is also easy to construct versions of this model that give a large electric dipole moment for electron, $D_e$. All these combine with the most recent bound on the charged Higgs boson mass from collider experiments to give a very tight constraint on the charged Higgs models of CP violation.

CP violation mediated by flavor conserving neutral Higgs bosons had been more or less ignored for a long time. The interest was renewed lately due to the recent experimental and theoretical progress which we shall review later. There are also many modern versions of the neutral Higgs CP violating models with basically the same backbone mechanism.[11, 12]

In 1976, Mohapatra and Pati[16] proposed to extend the gauge sector of the standard model to incorporate CP violation. They added the right handed gauge interactions to the left handed one in the standard model. The simplest version made use of the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. It made parity invariance possible at high energy[18]. The new gauge interaction implies a right-handed charged current which can be CP violating. This type of CP violating mechanism is milliweak in character. Therefore it is nontrivial to reproduce the superweak phenomenology which is so far consistent with experiments. If the mixing between the left-handed and right-handed currents can be ignored, then it can be shown that there are two CP violating phases in the theory if there are only two generations of fermions. In which



case, it was shown that the theory has a so called "iso-conjugate" relation which guarantees that the superweak result in the kaon system is reproduced[16, 17]. Unfortunately this is no longer the case when there are more than two generations because the existence of other phases in the theory. In general, in the $N$ generation case there are $N^2 - N + 1$ CP violating phases[19, 20, 21, 22] including the K-M phases. It was later shown that, among these phases, the complex coupling, $\eta_{LR}$ associated with the mixing between the left-handed and the right-handed charged currents can also give rise to very interesting CP phenomenology. This mixing is nonzero in most of the models. In fact, if one suppresses the K-M phases by implementing spontaneous CP violation, the phase of $\eta_{LR}$ becomes the leading contribution to the CP phenomenology[19] of the theory. This source of CP violation also has the character that it does not rely on generational mixing. Therefore, for the flavor neutral processes like $D_e$ or $D^c_G$, the heavy generation alone can give a contribution.

Many more CP violating mechanisms and associated models have been proposed in the literature since these early attempts. Among them the most interesting ones are the supersymmetric models[23, 24, 25, 26, 27]. In these models, there is typically more than one mechanism of CP violation. Generically, the CP violating sources are combinations of squark mixings and majorana masses of neutralinos. We shall illustrate some of these mechanisms in detail later.

Other interesting models include a host of superweak models[28]. They either contain flavor changing neutral gauge bosons, originating from some horizontal gauge symmetry, or flavor changing neutral Higgs bosons. For the gauge boson case, the intrinsic mechanism is not too different from the gauge boson mixing mechanism in the left-right models. For the neutral Higgs case, the mechanism is also similar to the scalar-pseudoscalar mixings in the neutral Higgs models of CP violation.

There is another class of models with mirror fermions or vectorial fermion pairs[29]. The simplest example of the latter class is given in the next section. In many grand unified models mirror fermions are needed for anomaly cancellation. In $E_6$ type grand unified models additional vectorial quarks and leptons are automatically included in the 27 dimensional representation. The mirror fermions give rise to right-handed interactions for the quarks and leptons and, as a result, its mechanism of CP violation is a lot like the left-right models.

There are also models with lepto-quark scalars[30] which are similar to the charged Higgs mixing mechanism with exotic Yukawa couplings. Still another interesting class are the models with the technicolor[31]. It is not easy to discuss such models because CP violation is closely tied with the flavor changing interaction which is the most uncertain and model dependent sector of technicolor models. For this reason not much work has been done in this direction. In any case, the simpler models of such type seem to be facing serious experimental challenge at the moment[32].

I shall divide the content of this review into two parts. The first part reviews the general concept of CP violation in gauge theory and many of the most popular gauge models of CP violation. More emphasis is put on the qualitative and conceptual aspects of CP violating mechanisms. In part II, I will review the most recent developments in CP violation. Due to space and time limitations, I shall confine myself to the subject of flavor neutral CP violation. Specifically, I shall deal with quantities like the electric dipole moment(EDM) of the neutron, $D_n$, the EDM of quarks, $D_q$, the chromo-EDM of quarks, $D^c_q$, the chromo-EDM of gluons, $D^c_G$, the EDM of the electron, $D_e$, and the EDM of the W boson, $D_W$.

# Part I. Mechanisms of CP Violation in Gauge Theory

## A. Sources of CP violation in field theory

The sources of CP violation in field theory can be classified into the following categories:

(I) Explicit breaking

As a result of the CPT theorem in field theory, CP violation necessarily implies T (time reversal) violation. Under the antiunitary transformation of time reversal, every field transforms into its complex conjugate, i.e. $\psi \to (\delta_\psi)\psi^\dagger$ where $\delta_\psi$ is a phase factor to be determined by the Lagrangian. Therefore the nonhermitian operators in the Lagrangian transform like $O \to (phase)O^\dagger$. Since the general form of the Lagrangian is

$$\mathcal{L} = \sum_i (C_i O_i + C_i^* O_i^\dagger), \tag{1}$$

the T-, or CP-violation will be reflected in the complex coupling constants in the Lagrangian. However not all complex coupling constants translate into genuine CP violation. The subtlety lies in the phase freedoms, $\delta_\psi$, that the theory has at its disposal. For a complex field, the free Lagrangian is invariant under the redefinition of its complex phase. We are free to redefine its phase without changing its identity. Therefore many of the complex coupling constants can be easily absorbed into these redefinitions. CP is broken only if one can not redefine the complex fields in the theory such that all the $C_i$ are real. For a massless fermion, we have an even larger $U(1)_L \times U(1)_R$ chiral symmetry for this purpose. For light fermions it is often useful to treat the mass as a perturbation and treat it on the same footing with all the other coupling constants. In that sense, it is not very important to use the mass eigenstates to do the analysis. One may as well use the basis in which the couplings are simplest to visualize. For the CP analysis, it can be useful even to do the same thing for heavy fermions just to make the qualitative properties more transparent.

Note that every redefinition one does affects all the terms in the Lagrangian which involve the redefined fields. The complexity of the couplings can easily be shifted from one term to another through these redefinitions. Therefore to make sure CP is broken, one has to check the reality of many coupling constants (including masses) in $\mathcal{L}$ simultaneously. Sometimes this freedom to shift the phases can be used to simplify the analysis of CP violating phenomenology by placing the complex phases strategically. As an example, the simplest CP violating toy Lagrangian one can write down involves a vectorial fermion, $\psi_L, \psi_R$, and a real scalar field $\phi$. The relevant terms in $\mathcal{L}$ are

$$m\bar{\psi}_L\psi_R + a\bar{\psi}_L\psi_R\phi + h.\ c. \tag{2}$$

Such CP violation can be easily constructed for the extra down-type quarks or leptons in an $E_6$-type unification models or any model with mirror fermions. The two coupling constants, $m$ and $a$, can not be made real simultaneously. Complexity can be assigned to either $a$ or $m$ when analyzing the CP phenomenology. Usually it is helpful to make the smaller parameter complex for obvious reasons.

One usually classifies the explicit breaking of symmetry into hard and soft breaking depending on whether the symmetry breaking terms are of dimension four or smaller. In field theory, to have a coupling constant to absorb every divergence in the theory, one likes to include all the possible interactions consistent with the required symmetry up to a certain dimension $d \leq 4$. The soft breaking has the advantage that there are less parameters in the theory and the induced dimension 4 breaking terms will be finite and calculable. For example, the simple Lagrangian in Eqn.( 2) should be considered soft breaking because if one set the mass term to zero then CP is a good symmetry. An example of the hard CP breaking is represented by the standard (Kobayashi-Maskawa) model of CP violation in which CP is broken explicitly in the dimension four Yukawa couplings already even before the gauge symmetry breaking.

(II) Spontaneous breaking
Even if the Lagrangian one starts out with is CP conserving, one can still break CP through spontaneous symmetry breaking. In a theory in which the vacuum expectation values(VEV's) are complex either because the Higgs bosons involved are complex or the condensates are complex, it can be written as $\langle H_i^0 \rangle = v_i e^{\theta_i}$. The complex phase is a signal of CP violation. However for consistency one has to make sure that the ground state of the Higgs potential, $V(H_i)$, allows nontrivial $\theta_i$. In addition, one has to check to see if the phase can not be rotated away by using the symmetry in the theory. For example, the single complex Higgs doublet in the standard model can not be used to implement spontaneously CP violation because the phase can be rotated away by the global hypercharge symmetry. Another example is the Weinberg model of CP violation with natural flavor conservation that will be discussed in more detail later. In that model due to the discrete symmetry one imposed on the theory to eliminate the flavor changing neutral currents, spontaneous CP violation can not be implemented even with two Higgs doublets. The symmetry allows only one nonhermitian term in the Higgs potential. As result the ground state of the potential does not allow nontrivial phases for VEV's.

All the symmetry breaking terms generated after the symmetry is broken will be proportional to the VEV. Since the vacuum expectation value has the dimension of mass, the symmetry breaking can be considered to be of the soft breaking type. In a spontaneous breaking theory, one usually imposes CP symmetry on the starting



Lagrangian. As a result, it reduces the number of independent complex phases in the theory and thereby organizes the CP phenomenology of the theory and increase its predicting power. One should note that there is usually more than one way to impose CP symmetry in the theory. The arbitrariness is related to the phase redefinition freedom we mentioned before. The simplest way is to fix a particular phase convention and then assume that all the coupling constants in this basis are real.

(III) Nonperturbative CP violation.
As a result of the existence of the instanton solution, the gauge invariant $\theta F_a^{\mu\nu}\tilde{F}_{\mu\nu}^a$ term has important physical effects even though it can be written as a total derivative. It is a P- and T-violating interaction. The effect of this term will be covered in detail in K. Choi's talk here. A very interesting aspect of it is that even though this term can be written as a total derivative, it can be induced through the renormalization group evolution[33]. I shall not elaborate further on this term.

## B. Left-handed Current Mechanism

In this model the CP violation can be placed in the left handed current interaction. The simplest of which is the three generational standard model. The relevant interactions before symmetry breaking are

$$\begin{aligned} -\mathcal{L}_W &= \frac{g}{\sqrt{2}}\bar{U}_L^i \gamma^\mu D_L^i W_\mu^\dagger + h.\,c. \\ -\mathcal{L}_Y &= h_{ij}\bar{Q}_L^i U_R^j \tilde{\Phi} + f_{ij}\bar{Q}_L^i D_R^j \Phi + h.\,c. \end{aligned} \tag{3}$$

where $Q_L$ and $\Phi$ are quark and Higgs doublets,

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{4}$$

$$\tag{5}$$

and $\tilde{\Phi} = i\tau_2\Phi^*$. The complex Yukawa couplings, $h_{ij}$, $f_{ij}$, are the origin of CP violation. $h$ and $f$ can be the most general N by N complex matrices for the $N$ generation model. However, not all complex parameters in $h$ and $f$ correspond to physical CP violation. We can use the large amount of freedom in the redefinition of the phases of the complex fields $U_i$ and $D_i$ to absorb a lot of them.

After the symmetry breaking $\langle\phi^0\rangle = \frac{v}{\sqrt{2}}$, and the quark masses are $(M_u)_{ij} = h_{ij}\frac{v}{\sqrt{2}}$ and $(M_d)_{ij} = f_{ij}\frac{v}{\sqrt{2}}$. These mass matrices can be diagonalized by the bi-unitary transformations: $(M_u)_{ij} = V_L^{u\dagger} D_u V_R^u$, $(M_d)_{ij} = V_L^{d\dagger} D_d V_R^d$. The matrices $V_R^u$ and $V_R^d$ drop out from the Lagrangian because there is no right-handed current in the theory. Only one combination, $\tilde{K} = V_L^u V_L^{d\dagger}$, remains in the Lagrangian. Therefore, in the end, besides the mass terms we have only the left-handed charged current interactions

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}}\tilde{K}_{ij}\bar{u}'^i_L\gamma^\mu d'^j_L W^\dagger_\mu + h.\ c. \tag{6}$$

where $u'$ and $d'$ are mass eigenstates of quarks. The unitary matrix $\tilde{K}$ in general has $\frac{N(N-1)}{2}$ angular parameters and $\frac{N(N+1)}{2}$ phase parameters. However besides the term in $\mathcal{L}_W$, the Lagrangian is still invariant under the redefinition of the phases of $u'^i_L$ and $d'^i_L$. We can use the quark phases to remove $2N-1$ phases from $\tilde{K}$( -1 because among the $2N$ redefinitions one of them does not affect $\mathcal{L}_W$ at all). Therefore in an $N$ generation model, there are $\frac{(N-1)(N-2)}{2}$ CP violating phases. For three generations, there is exactly one phase. From now on we shall restrict ourselves to this case. The resulting matrix with one complex phase is called CKM matrix, $K$.

Without CP violation, the general form of $K$ can be written as a product of three two dimensional rotations. Call $R(\theta_i)$ the rotation in $j-k$ plane where $(ijk)$ are some cyclic permutations of (123). The general CP conserving (orthogonal) $K$ matrix can be written as $K = R(\theta_i)R(\theta_j)R(\theta_k)$ where $i,j,k \in \{1,2,3\}$ with $i \neq j$ and $j \neq k$. All the known parametrizations of the CKM matrix can be written in this form with proper diagonal unitary matrices sandwiched between different $R$'s. For example the original parametrization by Kobayashi and Maskawa[4] can be written as

$$\begin{aligned}
\mathbf{V}_{CKM} &= \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta_{KM}} & c_1c_2s_3 + s_2c_3e^{i\delta_{KM}} \\ -s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta_{KM}} & c_1s_2s_3 - c_2c_3e^{i\delta_{KM}} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix}\begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -e^{i\delta_{KM}} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} \\
&\equiv R_2 \cdot R_1 \cdot P_{KM} \cdot R_3
\end{aligned} \tag{7}$$

where $c_i = cos\theta_i, s_i = sin\theta_i$ and $\delta_{KM}$ is the CP-violating phase.



From this parametrization, one can show that CP violation disappears if any of the $c_i$ or $s_i = 0$. For $s_2$ or $s_3 = 0$, the diagonal phase matrix, $P_{KM}$ can be moved to the rightmost or the leftmost position. The CP violating phase can then be absorbed into the redefinition of the phases of the quarks. For $c_2$ or $c_3 = 0$, the relation

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -e^{i\delta_{KM}} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -e^{i\delta_{KM}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{8}$$

implies again that the phase matrix can be removed. For $s_1 = 0$ all three R matrices reduce to 2 by 2 matrices which we know has no genuine CP violating phase. In fact the most general 2 by 2 unitary matrix can be written as

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta_1} \end{pmatrix}\begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}\begin{pmatrix} e^{i\delta_2} & 0 \\ 0 & e^{i\delta_3} \end{pmatrix} \tag{9}$$

Clearly the phases $\delta_i$ are not physical. For $c_1 = 0$,

$$\begin{aligned}
& R_2 \cdot R_1 \cdot P_{KM} \cdot R_3 \\
&= R_1 \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -e^{-i\delta_{KM}} & 0 \\ 0 & 0 & 1 \end{pmatrix} R_3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -e^{i\delta_{KM}} & 0 \\ 0 & 0 & -e^{i\delta_{KM}} \end{pmatrix} \\
&= \begin{pmatrix} -e^{-i\delta_{KM}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} R_1 \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix} R_3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -e^{i\delta_{KM}} & 0 \\ 0 & 0 & -e^{i\delta_{KM}} \end{pmatrix}
\end{aligned} \tag{10}$$

Therefore we expect the the CP violating effect to be proportional to $c_1c_2c_3s_1s_2s_3$ $\cdot sin\delta_{KM}$.

The same argument can be easily applied to any other parametrization of CKM matrix because they are all of the form of $R(i)R(j)R(k)$ with a phase matrix sandwiched between the $R$'s. For example, the parametrization used in the recent particle data books can be written as

$$\mathbf{V}_{CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \equiv \tag{11}$$

(12)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} \tag{13}$$

where $s_{12} = \sin\theta_{12}$, etc. One can also adopt the convention where all three mixing angles lie in the first quadrant and the phase is restricted to the interval $0 < \delta < \pi$. This parametrization was first proposed by Chau and Keung[34]. It has the advantage that the CP violating phase is automatically multiplied by the smaller mixing angle. Experimentally, $s_{13}$ was found to be very small. In addition, the upper triangle of the CKM matrix is better measured experimentally. In this parametrization, these measurements can be translated into limits on angles more directly. For example, the experimental limit on $V_{ub}$ implies $c_{13} = 1 + O(10^{-3})$. This in turn implies that the well measured $V_{us}$ and $V_{cb}$ determine the angles $s_{12}$ and $s_{23}$ directly. Since all the $s_{ij}$'s are small, the CKM matrix can be approximated by

$$\mathbf{V}_{CKM} \simeq \begin{pmatrix} 1 & s_{12} & s_{13}e^{-i\delta} \\ -s_{12} & 1 & s_{23} \\ V_{td} & V_{ts} & 1 \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{14}$$

This simplified (Wolfenstein) parametrization[35] is extremely useful for doing phenomenological analysis even though it violates unitarity slightly.

If any of the two up-type quarks have the same mass then one of the $R(\theta)$ matrices can be reduced to one of the trivial forms corresponding to $c = 0$ or $s = 0$, and CP violation disappears. The same thing also happens when any two of the down-type quarks have the same mass. As a result we also expect CP violation to be proportional to $U \equiv (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \simeq m_t^4 m_c^2$ and $D \equiv (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \simeq m_b^4 m_s^2$. Therefore any CP violating amplitude has to be



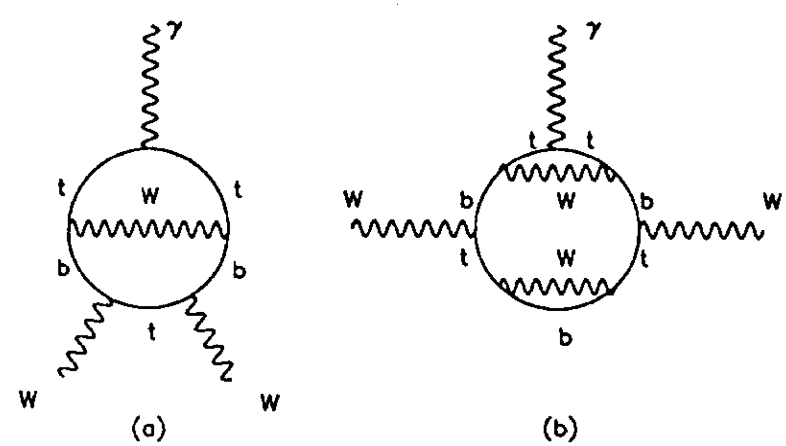


Figure 1: A typical (a)two loop or (b) three loop contribution to the electric dipole moment of the W in the K-M model.

proportional $U \cdot D \cdot J$ where $J$ is either $c_1c_2c_3s_1^2s_2s_3 sin\delta_{KM}$ in the K-M parametrization, or $c_{12}c_{13}c_{23}^2 s_{12}s_{13}s_{23} sin\delta$ for the parametrization in Eqn( 41). The square on $s_1$ or $c_{13}$ is included such that $J$ can be written in the parametrization-independent form of (up to a sign)

$$J = \pm \mathrm{Im}(V_{ij}V_{lk}V_{ik}^*V_{lj}^*) \tag{15}$$

Absolute value of $J$ is $\leq 10^{-4} sin\delta$. Together with $U$ and $D$, they can be written in a compact form as

$$\mid UDJ \mid = \mid \mathrm{Im}(\det[M_d M_d^\dagger, M_u M_u^\dagger]) \mid \tag{16}$$

This analysis can be very useful in making phenomenological analysis. For example, one can use it to estimate the magnitude of electric dipole moment(EDM), $D_W$, of the W gauge boson in the K-M model[61]. This operator will be discussed in more detail in Part II when we talk about recent developments. It is easy to convince oneself that there is no contribution at the one loop level. To get CP violation in this model, one has to get all three generations of fermions involved in the process. Since there are no fermions in the external lines, the fermions can get involved only in the loop. At one loop level, there are clearly not enough fermion lines to do that. It will take at least a two loop process to contribute to $D_W$. A typical two loop diagram is given in Fig. 1(a). Assuming that the two loop contribution is nonzero, one can estimate $D_W$ to be

$$|D_W| \leq \left(\frac{g^2}{8\pi^2}\right)^2\left(\frac{e}{2M_W}\right)|J|\frac{m_t^4 m_b^4 m_s^2 m_c^2}{M_W^{12}} \leq 10^{-38} e - cm, \tag{17}$$

for $m_t \sim M_W$. Experimentally, $|J| \leq 10^{-4}$. In fact it is completely possible that the two loop diagrams may sum up to zero. (One is reminded that Shabalin[37] proved that the EDM of quarks in the K-M model is zero at the 2 loop level after many others had made nonzero estimates). In that case one will have to go to the three loop level as in Fig. 1(b). We shall not bother to investigate that because at the two loop level the numerical prediction of $D_W$ is already smaller than many other models of CP violation by at least 16 orders of magnitude.

Another example can be found in a recent calculation by Hoogeveen[38] of the EDM of the electron in the K-M model. He claimed to obtain a nonzero result after a tedious three loop calculation. It is not surprising that it takes at least three loops to get a nonzero result since the contribution is induced through an effective vertex which represents the electric dipole moment of the W. The numerical result is summarized as[38]

$$D_e = -1.7 \times 10^{-38}\left(\frac{m_t}{100GeV}\right)^2\left(\frac{J}{10^{-4}}\right)e - cm. \tag{18}$$

which is indeed very small as expected.

The phenomenology of this "standard" model of CP violation is reviewed in detail in C.S. Kim's talks in this Symposium. I shall not repeat them here. Let me emphasize the lesson one should learn from the above analysis. The K-M model is a milliweak type model of CP violation. However it behaves like a superweak model because of the $UDJ$ factors discussed earlier combined with the suppression factor due to the unitarity constraint of the quark mixing matrix. This is the magic of the K-M model. On the other hand, one can also argue that since all the experimental data indicate that the origin of the CP violation is likely to be of the superweak type, one would like to have a model of CP violation which is "naturally" superweak. The fact that superweak predictions in K-M model are achieved only because the above "factors" are accidentally small may be considered as an unsatisfactory feature.



# C. Higgs Exchange Mechanism

Here we shall use the Weinberg-Branco[6, 10] model of spontaneous CP violation as the example for illustration. This model contains both the neutral and charged Higgs exchange mechanisms and is constrained most severely by the experiments. The model assumes that there is no tree level flavor changing neutral current and CP is broken only spontaneously. To have spontaneous CP violation, at least two Higgs doublets, $\Phi_i$, are needed with

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} \tag{19}$$

To eliminate the flavor changing couplings of neutral Higgs bosons, one imposes a discrete symmetry $D$

$$D : \begin{cases} \Phi_2 & \rightarrow -\Phi_2 \\ U_R & \rightarrow -U_R \end{cases} \tag{20}$$

so that $\Phi_2$ couples only to $U_R$ while $\Phi_1$ couples only to $D_R$.

$$-\mathcal{L}_Y = h_{ij}\bar{Q}_L^i U_R^j \tilde{\Phi} + f_{ij}\bar{Q}_L^i D_R^j \Phi \; + \; h.\ c. \tag{21}$$

Note that a sufficient condition for elimination of flavor changing neutral currents is that each flavor of fermions obtains its mass from only one source or from the VEV of only one Higgs field[7]. With this discrete symmetry, the Higgs potential has only one nonhermitian term, $c(\Phi_1^\dagger\Phi_2)^2$. The VEV's of the $\Phi_i$'s, $\langle\phi_i^0\rangle \equiv \lambda_i \equiv v_i e^{i\theta_i}$, are in general complex. One can use the $U(1)_Y$ symmetry to redefine one of the $\theta$, say $\theta_1$, to be zero. The coupling $c$ can be made real by redefining $\Phi_1$ and $D_R$. Therefore the vacuum expectation value of the Higgs potential becomes $A + cB\cos\theta_2$ where $A$, $B$ are functions of $v_i$. Clearly the ground state corresponds to $\theta = n\pi$ which are CP invariant vacua. As a result, when the Higgs potential is minimized the ground state is automatically CP invariant. (Note that this is true of any Higgs potential with only one nonhermitian term). This implies we need at least three doublets to get spontaneous CP violation.

Another consequence of imposing CP symmetry before gauge symmetry breaking is that the Yukawa couplings will become real and the left-handed current will contain no CP violation at the tree level no matter how many generations of fermions we have. That is, *there is no* K-M mechanism in the model. As a result one has to explain the CP violation in kaon decay solely by the Higgs exchange mechanism. This allows a cleaner test of the theory and can be considered a nice feature of the spontaneous CP breaking model.

For the minimal case of using three doublets, one has two physical charged Higgs and 5 physical neutral (real) Higgs. Because of the disappearance of the flavor changing neutral current, the neutral Higgs sector can contribute to the CP violating $\epsilon$ only at the two loop level. Therefore, charged Higgs sector will be solely responsible for explaining $\epsilon$. As a result the charged Higgs can not be too heavy. For this reason we shall discuss the charged Higgs sector first. After diagonalizing the mass matrices, the Yukawa interactions of the charged Higgs become

$$-\mathcal{L}_Y = -\frac{1}{v_1}\bar{d}^i_L[O^c D_u]_{ij} u^j_R \phi_2^{-\prime} + \frac{1}{v_2}\bar{u}^i_L[O^c D_d]_{ij} d^j_R \phi_1^{-\prime} + h.\ c. \tag{22}$$

where $\phi_i' \equiv \phi_i e^{-i\theta_i}$, and $O^c$ is the usual CKM matrix with the KM phase $\delta = 0$. $D_u, D_d$ are diagonal quark mass matrices. The Higgs self interactions give rise to a very complicated mass matrix for charged Higgs bosons. However it can be shown that the unitary matrix, $U_H$, that relates weak eigenstates $\phi$'s to mass eigenstates $H_i$'s

$$(H_3^+, H_1^+, H_2^+)^T = U_H^\dagger(\phi_3', \phi_1', \phi_2')^T \tag{23}$$

can be written in the same expression as the CKM matrix in Eqn.( 22). This is because the content of the Goldstone boson $G^+ \equiv H_3^+$ is determined by symmetry breaking completely as

$$\begin{aligned} G^+ &= \frac{1}{V}(\langle\phi_1^0\rangle\phi_1^+ + \langle\phi_2^0\rangle\phi_2^+ + \langle\phi_3^0\rangle\phi_3^+) \\ &= \frac{1}{V}(v_1\phi_1^{+\prime} + v_2\phi_2^{+\prime} + v_3\phi_3^{+\prime}) \\ &= s_1c_3\phi_1' + s_1s_3\phi_2' + c_1\phi_3' \end{aligned} \tag{24}$$

where $V^2 = v_1^2 + v_2^2 + v_3^2 = (2\sqrt{2}G_F)^{-1}$ and $s_i$'s, $c_i$'s are sines and cosines of the appropriate Higgs mixing angles. This equation also defines $s_1$ and $s_3$ through $c_1 = \frac{v_3}{V}$ and $t_3 = \frac{v_2}{v_1}$. Therefore in the charged Higgs sector there is only one CP violating phase.



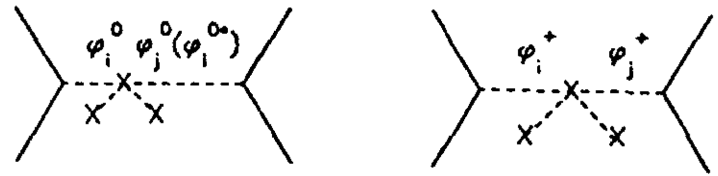


Figure 2: Diagrams representing neutral or charged Higgs exchange with CP violating mixing.

The CP violating effects are contained in the Higgs boson exchange interactions. The quark coupling (and the probably lepton coupling too) to the Higgs is most simply expressed in the weak eigenstates $\phi_i$ as in Eqn( 21). Therefore the effect of the charged Higgs exchange can be expressed in terms of the imaginary part of

$$\begin{aligned} A_{12} &\equiv \frac{\langle\phi_1^-\phi_2^+\rangle}{v_1v_2} = \sum_n \frac{U_{2n}^{H*}U_{n1}^H}{v_1v_2}\frac{1}{q^2 - M_{H_n}^2} \\ &\equiv -\sum_n \frac{\sqrt{2}G_F Z_i^n}{q^2 - M_{H_n}^2} \equiv 2\sqrt{2}G_F\sum_n \frac{a_n}{q^2 - M_{H_n}^2}. \end{aligned} \tag{25}$$

*For neutral Higgs models, They can be parametrized as*

$$\begin{aligned} A_i &= \frac{1}{\lambda_i^2}\langle\phi_i^0\phi_i^0\rangle = -\sum_n \frac{\sqrt{2}G_F Z_i^n}{q^2 - M_{H_n}^2}, \\ A_{ij} &= \frac{1}{\lambda_i\lambda_j^*}\langle\phi_i^0\phi_j^{0*}\rangle = -\sum_n \frac{\sqrt{2}G_F Z_{ij}^n}{q^2 - M_{H_n}^2}, \\ \tilde{A}_{ij} &= \frac{1}{\lambda_i\lambda_j}\langle\phi_i^0\phi_j^0\rangle = -\sum_n \frac{\sqrt{2}G_F \tilde{Z}_{ij}^n}{q^2 - M_{H_n}^2}. \end{aligned} \tag{26}$$

Eqn( 25, 27) can be represented by the diagrams in Fig. 2. The sums above do not include the contributions from the neutral Goldstone boson as it will not participate in the CP violating amplitudes. Such exemption makes these definitions independent of the gauge parameter $\xi$ in the $R_\xi$ gauge. Most of the time in this article, it is assumed that the effect of the lightest Higgs dominates the sum and therefore we ignore the index $n$ of the $Z$. By definition, $A_{ij}$ is hermitian and $\tilde{A}_{ij}$ is symmetric. Note that the same formula can be used for an arbitrary number of Higgs bosons.

For the two doublet case there are 4 CP violating parameters in Eqn( 27). However, not all of them are independent. In the unitary gauge,

$$\mathrm{Im}(\sum_n \langle\phi_n\rangle^* \phi_n) = 0. \tag{27}$$

By requiring

$$\mathrm{Im}\left[(\sum_i \langle\phi_i\rangle^* \phi_i)(\langle\phi_j\rangle^* \phi_j + \langle\phi_j\rangle \phi_j^*)\right] = 0 \tag{28}$$

for each $j$, one is lead to the unitarity gauge condition

$$\mathrm{Im}\ A_i = \sum_{k\neq i}^{N} |\frac{\lambda_k}{\lambda_i}|^2 \left(\mathrm{Im}\ A_{ik} - \mathrm{Im}\ \tilde{A}_{ik}\right) . \tag{29}$$

This equation is gauge independent. In particular, for $N$ Higgs boson doublets we have $\mathrm{Im}\ A_1 = -|\lambda_1|^{-2} \sum_{k=2}^{N} |\lambda_k|^2 (\mathrm{Im}\ A_{k1} + \mathrm{Im}\ \tilde{A}_{k1})$.

For $N = 2$, it implies that there are only two independent CP violating parameters for this case. In Ref.[12] Weinberg used this equation to derive an upper bound on the CP violating parameter Im $Z_2$. For multi-Higgs doublets models, there are more than two CP violating mixings. For the charged Higgs sector, the CP violating parameters are $a_n$ which satisfy the unitarity constraint $\sum_n a_n = 0$.

Before we get into the thick of phenomenological analysis, I like to re-emphasize a basic feature of the Higgs models of CP violation. In order to get CP violation from the exchange of a Higgs boson, it is necessary for the fermions that couple to the Higgs boson in the diagram to be massive. For the neutral Higgs case, it can be seen in the simple Lagrangian in Eqn. 2. There, if the fermion is massless, the CP violation disappears. This is because of the enhanced chiral symmetries of the free Lagrangian when the fermion is massless. These symmetry can be used to find a CP symmetry for the interaction when was not possible if there is an extra mass term. The phenomenological consequence of this is that the diagrams that induce CP violating quantities, like the chromo-electric dipole moment of gluon that we will discuss in Part II, will have to have fermion mass insertion in the diagram in order to become CP violating. For small fermion mass, we can conclude without calculation



that the result will be proportional to at least one power of efrmion mass. This rule has to be used carefully though. For example, if one is calculating the induced electric dipole moment of a fermion $f$, one does not necessarily expect to have $m_f$ for the reason that we will discussed in detail in *Example* 1 in Section I.E. For the charged Higgs case, one can see this in the Lagrangian

$$m_1\bar{\psi}_{1L}\psi_{1R} + m_2\bar{\psi}_{2L}\psi_{2R} + a\bar{\psi}_{1L}\psi_{2R}\phi^+ + b\bar{\psi}_{1L}\psi_{2R}\phi^+ + h.\ c. \tag{30}$$

where $\psi_1$ and $\psi_2$ differ in their charges by one unit. Three out of the four parameters in the Lagrangian can be made real by proper redefinition of phases. That mean if any of the parameters is zero, there will be no CP violation.

From now on we shall concentrate on the charged Higgs sector in this section. This is the sector that has been developed more extensively. We will get back to the neutral Higgs sector when we review recent developments.

The phenomenology of the model was recently reviewed by Cheng[15]. We shall simply summarize the result pedagogically here. The main constraint on the model comes of course from the only CP violation we have observed in nature, the $\epsilon$ parameter. The CP violation in kaon decay to two pions can be parametrized with $\epsilon$ and $\epsilon'$. $\epsilon$ can be defined as

$$\begin{aligned}
\epsilon &= \frac{1}{\sqrt{2}}(\rho + \eta_0)e^{i\pi/4} \\
\rho &\equiv \frac{\mathrm{Im}M_{12}}{2\mathrm{Re}M_{12}} \equiv \frac{m'}{\Delta m} \\
\eta_0 &\equiv \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}
\end{aligned} \tag{31}$$

where $M_{ij}$ is the neutral kaon mass matrix in the basis of $K^0 - \bar{K}^0$; $A_0$ is the isospin-zero amplitude of $K \to 2\pi$. This formula is true in the phase convention in which the isospin-two amplitude, $A_2$, is real. This can always be done because we are free to redefine the strange quark field by a phase. However, when one deals with a specific theory the convention may have already been specified by Lagrangian. Therefore one should be careful to make sure one uses a consistent basis. It is possible to write $\epsilon$ in a basis independent[17, 39] form but we shall not get into that here.

In this model $\mathrm{Im}M_{12}$ may have both short and long distance contributions. The short distance contributions are due to the box diagrams with one or two charged

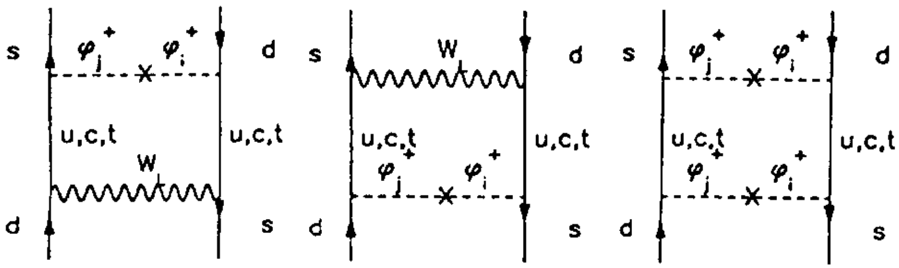


Figure 3: Box diagrams with one or more charged Higgs bosons exchanged.

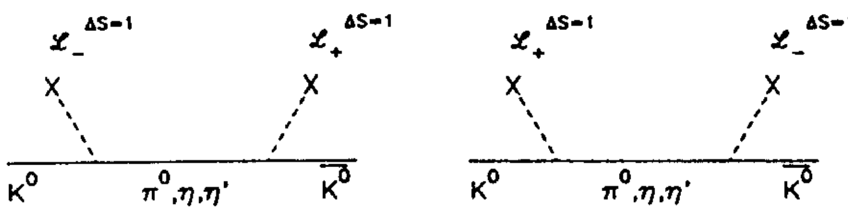


Figure 4: Long distance dispersive contribution to $K - \bar{K}$ mixing.

Higgs exchanged as shown in Fig. 3. The long distance dispersive contribution can arise from the $\pi, \eta$ and $\eta'$ poles between kaon and $\pi$ as shown in Fig. 4. It was argued[9] that for this model to be viable, the long distance contribution should be larger than the short distance contribution. This is indeed confirmed by many estimates[18, 40]. To calculate the dispersive contributions one needs the operator for the $\Delta s = 1$ CP violating transition. This is done by calculating the so-called Higgs Penguin diagram in Fig. 5.[41, 42] The internal loop is dominated by the charm quark contribution. The remaining matrix elements are calculated by chiral perturbation theory. One notes that $\eta, \eta'$ provide contributions as important as $\pi$. The result can



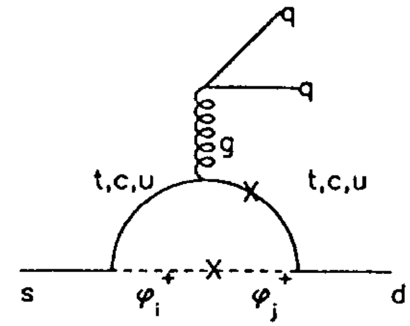


Figure 5: Higgs penguin diagram contributing to the $\Delta s = 1$ process in the charged Higgs models.

be translated into the constraint

$$\frac{\mathrm{Im}(a_1)}{M_H^2}\left(\ln\frac{m_H^2}{m_c^2} - \frac{3}{2}\right) = (.024 \sim .027) GeV^{-2} \tag{32}$$

where $a_1$ is defined in Eqn( 25) and $M_H$ is the mass of the lightest Higgs, $H_1$. We have also used the experimental value for $|\epsilon| = 2.27 \times 10^{-3}$ and the fact that $\eta_0$ has to be much smaller than $\rho$ so that $\epsilon'$ is not too large. With the CERN collider LEP data pushing the mass of the charged Higgs to be $\geq 45 GeV$, $\mathrm{Im}(a_1)$ has to be larger than 9.2 which is not very natural for a quantity which normally expected to be of order one. From Eqn( 25), $a_n = U_{2n}^{H*} U_{n1}^{H} (V^2/v_1 v_2)$.

The model does tend to give a large value for $\epsilon'$ which is defined to be

$$\frac{\epsilon'}{\epsilon} = -\frac{1}{22}\left(\frac{\eta_0}{\rho + \eta_0}\right). \tag{33}$$

To derive the consequence of the experimental upper bound, one needs to calculate $\eta_0$. The main contribution also arises from the Higgs penguin diagrams[41, 42]. However in this case we need the matrix element between $K^0$ and $2\pi$. The first estimate[41] of this ratio gave 1/22, and as a result the model was considered ruled out for a while. Later it was pointed out[42] that the value for this matrix element is very uncertain because in the chiral perturbation theory the leading contributions to this matrix element cancelled. Therefore it is difficult to evaluate the matrix element

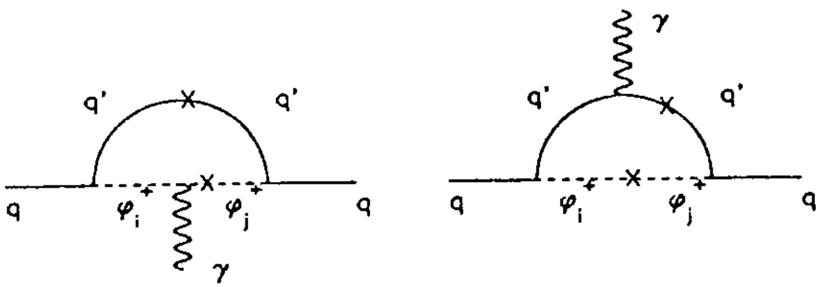



Figure 6: One loop Feynman diagrams contributing to the electric dipole moment of the quark in the charged Higgs models.

with good confidence in chiral perturbation theory. The most recent estimate[42] gives $\frac{\epsilon'}{\epsilon} \simeq -0.007$ which may be still too large if the limit by the E731 experiment[43] holds up. Given the uncertainty in this calculation, one should still investigate other types of phenomenology in this model.

Many mechanisms for the neutron electric dipole moment, $D_N$, in this model were considered in the literature. The simplest one is due to the electric dipole moment of quarks, $D_q$. The nonrelativistic quark model gives the relation $D_N = (4/3)D_d - (1/3)D_u$. In the one loop contribution in Fig. 6 CP violation requires the two Yukawa vertices to be correlated in such a way that at least one Yukawa coupling will be proportional to the external mass. This is because the two quarks at the Yukawa vertex, Eqn( 22), have to have different helicity and the coupling is proportional to the mass of the quark with right-handed helicity. To get CP violation, one needs to flip helicity one more time in the internal line so that the product of the couplings will be complex. This implies that only the mass part of the numerator of the internal propagator contributes. This is typically called a "mass insertion" in the literature. For internal quarks lighter than the Higgs, one can easily estimate the diagram to be

$$D_q = Q_q(2\sqrt{2}G_F \mathrm{Im}a_1)(4\pi)^{-2}m_{q'}\sum_{q'}K^2_{qq'}f(\frac{m^2_{q'}}{m^2_H}) \tag{34}$$



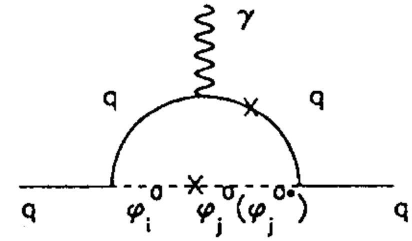



Figure 7: Feynman diagrams contributing to the electric dipole moment of the quark in the neutral Higgs models.

where $K_{qq'}$ is the CKM quark mixing matrix; $f(x)$ is of order one if $m_{q'}$ is of order $M_H$ and $f(x) \simeq c \cdot x\ln x$ if $x$ is very small. This is because after the required internal mass insertion the loop integration has logarithmic divergence when the internal quark mass is very small. The constant $c$ is evaluated in the explicit loop calculation to be -1/2. Therefore, $D_N \simeq (4/3)D_d = -9 \times 10^{-26}e - cm$ which is on the verge of being ruled out by the experiments.

One may wonder why there is no external quark mass dependence in this formula, since, as we just learned, there should be no CP violation in the diagram if the quark mass is zero. The answer is that for massless quark, the electric dipole moment in fact is not a CP violating quantity. This will be explained in more detail in *Example* 1 of Section I.E.

The neutral Higgs sector can also give rise to large $D_N$. In this case we can not make definite prediction because the sector is not constrained by the observed CP violation: $\epsilon$. One can always make $M_{H^0}$ larger to suppress its effect. This is because typically it takes at least a two loop process for the neutral Higgs to change strangeness by two units. Its contribution to $\epsilon'$ is negligible. The one loop contribution to $D_q$ is given in Fig. 7. Since the light neutral Higgs has to be flavor conserving, the diagram automatically requires three powers of light quark masses. Therefore it is very small. However, Anselm et.al[44] have pointed out another way the neutral

Higgs can contribute. At one loop level, the Higgs can couple to gluon through the heavy quark loop. The coupling can be either scalar or pseudoscalar. These one loop vertices combine with CP violating neutral Higgs exchange give rise to new contributions to $D_N$. Weinberg[6] pointed out that this mechanism can be interpreted as the result of an induced dimension 8 purely gluonic operator at the two loop level which can be represented by Fig. 21(a). A recent estimate[15] of this effect gives $D_N \simeq 2 \times 10^{-26} \cdot (100GeV/M_H)^2$. This can be quite close to the experimental limit if the neutral Higgs is $100GeV$ or lighter. We will have more to say about this dimension 8 operator in Part II.

## D. Right-Handed Current Mechanism

The simplest gauge group for this case is the G(LR) $\equiv SU(2)_L \times SU(2)_R \times SU(3)_C \times U(1)_{B-L}$. The standard Higgs multiplet used to break symmetry includes

$$\begin{aligned} \Phi &= \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \ : (2,2,1_c;0); \\ \Delta_L &\ : (3,1,1_c;2) \\ \Delta_R &\ : (1,3,1_c;2) \end{aligned} \tag{35}$$

where the numbers in the bracket indicate the representation of the multiplet under the group G(LR). $\Delta$'s are needed because $\Phi$ alone can break G(LR) only down to $U(1)_{L+R} \times U(1)_{B-L}$. In the literature, sometimes multiplets:

$$\begin{aligned} \chi_L &\ : (2,1,1_c;1) \\ \chi_R &\ : (1,2,1_c;1) \end{aligned} \tag{36}$$

were used instead of $\Delta$'s. The advantage of using $\Delta$'s is that their couplings to the neutrinos are such that, when they develop nonzero V.E.V.'s, $\langle \Delta_{L(R)} \rangle$ gives rise to majorana mass for $\nu_{L(R)}$. Since $\langle \Delta_R \rangle$ breaks $SU(2)_R \times U(1)_{B-L}$ symmetry down to $U(1)_{em}$ it is usually assumed to be much larger than $\langle \Phi \rangle$. Therefore one is naturally lead to large majorana mass for the right-handed neutrinos. This together with the Dirac masses of the order of the charged lepton masses leads to one very light neutrino



mass eigenstate per generation through the so called See-Saw mechanism[16]. We shall not get into more details in that direction. However we are going to use $\Delta$'s in our subsequent discussion even though in most cases it is not important for our purposes.

The fermions in the theory are in the representations

$$\begin{aligned} \Psi_L &\ : (2,1,1_c;1) \\ \Psi_R &\ : (1,2,1_c;1) \end{aligned} \tag{37}$$

for leptons and

$$\begin{aligned} Q_L &\ : (2,1,3_c;1) \\ Q_R &\ : (1,2,3_c;1) \end{aligned} \tag{38}$$

for quarks.

The charged current (gauge) interaction can be written as

$$\mathcal{L}_C = \frac{g_L}{\sqrt{2}} \bar{u}^i_L \gamma_\mu d^i_L W^{L\mu +} + \frac{g_R}{\sqrt{2}} \bar{u}^i_R \gamma_\mu d^i_R W^{R\mu +} + h.\ c. \tag{39}$$

and the Yukawa coupling interaction as

$$\mathcal{L}_Y = f_{ij} \bar{\Psi}_{iL} \Phi \Psi_{jR} + h_{ij} \bar{\Psi}_{iL} \tilde{\Phi} \Psi_{jR} + h.\ c. \tag{40}$$

where $\tilde{\Phi}$ is defined to be

$$\tilde{\Phi} = \tau_2 \Phi^* \tau_2 = \begin{pmatrix} \phi_2^{0*} & -\phi_2^+ \\ -\phi_1^- & \phi_1^{0*} \end{pmatrix} \ : (2,2,1_c;0). \tag{41}$$

The neutral components in $\Phi$ are arranged such that symmetry is broken down to $U(1)_{em}$ with charged $Q$ defined by $Q = T_{3L} + T_{3R} + (B-L)/2$ where $T_{3L(R)}$ is the third component of $SU(2)_{L(R)}$. The most general vacuum expectation values for $\Phi$ is

$$< \Phi > = \begin{pmatrix} \kappa e^{i\eta'} & 0 \\ 0 & \kappa' e^{i\eta} \end{pmatrix}. \tag{42}$$

This VEV breaks $SU(2)_L$ and gives mass to $W_L$. One can use the global $U(1)_{L-R}$ symmetry to rotate one of the phases to zero. We shall pick the phase $\eta'$ to be zero. $W_R$ gets a much larger mass from $\langle \Delta_R \rangle = V_R$. The mass matrix of the gauge bosons can be written as

$$(W_L^+, W_R^+) \begin{pmatrix} g_L^2 \frac{|\kappa|^2 + |\kappa'|^2}{2} & -g_L g_R \kappa \kappa' e^{i\eta} \\ -g_L g_R \kappa \kappa' e^{-i\eta} & g_R^2 (\frac{|\kappa|^2 + |\kappa'|^2}{2} + V_R^2) \end{pmatrix} \begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix}. \tag{43}$$

where we have assumed that $\langle\Delta_L\rangle = V_L$ is negligibly small. We shall refer the reader to the literature regarding the subtle point of how to do this naturally[18]. It will not be important for our purposes.

The mass eigenstates of the gauge bosons can be defined as

$$\begin{aligned} W_1^+ &= \cos\xi W_L^+ + e^{-i\eta}\sin\xi W_R^+ \\ W_2^+ &= -e^{+i\eta}\sin\xi W_L^+ + \cos\xi W_R^+ \end{aligned} \tag{44}$$

where $\tan 2\xi = 2g_L\kappa\kappa'/g_R V_R^2$ for $V_R \gg \kappa,\ \kappa'$. Note that the phase $\eta$ in the left-right mixing is a genuine $CP$ violating parameter. If one tries to absorb it into the redefinition of complex $W_R^\mu$ field, one finds that the phase will appear in front of the $g_R$ in the charged current interaction. Therefore this is a genuine relative phase between the left-handed and the right-handed currents.

For the N generation case, the charged currents can be written as

$$\mathcal{L}_C = \frac{g_L}{\sqrt{2}}\bar{u}_{Li}\gamma_\mu(\tilde{K}_L)_{ij}d_{Lj}W_L^{\mu+} + \frac{g_R}{\sqrt{2}}\bar{u}_{Ri}\gamma_\mu(\tilde{K}_R)_{ij}d_{Rj}W_R^{\mu+} + h.\,c. \tag{45}$$

where $\tilde{K}_{L(R)} = V^u_{L(R)}(V^d_{L(R)})^\dagger$ with $V^u_{L(R)}, V^d_{L(R)}$ defined in Section I.B. One can now do similar redefinition as in Section I.B. to remove phases from $\tilde{K}_L$. However once all the phase freedoms are used up in removing phases from $\tilde{K}_L$, $\tilde{K}_R$ will remain the most general $U(N)$ matrix. The $U(1)$ piece of the $U(N)$ can be identified as the phase of the left-right mixing $\xi$. Therefore the number of CP violating phases in the theory is $(N-1)(N-2)/2 + N(N+1)/2 = N^2 - N + 1$. For $N = 1$ we have exactly one CP violating phase which is the phase of left-right mixing.

One can try to eliminate some of these independent complex phases by imposing more symmetry on the Lagrangian. There are two popular symmetries in the literature for this purpose, one of them is the parity, conventionally defined as

$$LR: \begin{cases} W_L \leftrightarrow W_R; & \psi_{iL} \leftrightarrow \psi_{iR}; \\ \Delta_L \leftrightarrow \Delta_R; & \Phi \leftrightarrow \Phi^\dagger; \end{cases} \tag{46}$$

and the other one is the CP symmetry. If one imposes LR symmetry, the coupling matrices $f_{ij}, h_{ij}$ in Eqn( 40) will become hermitian. However that does not mean that the matrices will be hermitian because the VEV's are complex. CP symmetry



will imply that these matrices are real. A very interesting scenario is that when both LR and CP are imposed, the only origin of CP violation is in the VEV's. If one uses only one doublet this phase has to be the relative phase in Eqn( 42). In that case all the CP violating phases in $K_{L(R)}$ should be calculable in terms of quark masses, mixing angles and the vacuum phase $\eta$. In fact in such models, the left-right mixing phase is the only phase in $K_{L(R)}$ that is not suppressed by additional mass ratios. In fact, it is surprising that one naturally obtains small $\epsilon'/\epsilon$ under such very simple assumptions. This approach was started in Ref.[19] and was extended in many later works[20, 21, 22]. In this type of model and any other models in which the left-right mixing is nonzero, the CP violating phase in the left-right mixing plays a very important role. Even though experimentally this mixing is constrained to be quite small ($\leq 0.0055$) already.[45] Note the the left-right mixing is automatically small in the model because it is suppressed by the right-handed scale as can be seen in Eqn( 44).

There are of course other models which simply assume that left-right mixing is negligible. Due to limitations in space and time we shall not go into them further. Instead, we shall concentrate on the effect of CP violation due to left- right mixing. In that case, to simplify the discussion, one can assume that the quark mixing angles are negligible because they are not relevant to obtaining CP violation. Therefore we can even discuss one generation at a time. Before we get into the mess of it, let's first make sure that we understand our source of CP violation again. It is a good exercise to check that among the complex couplings: $g_L\bar{u}_L\gamma_\mu d_L W_L^{\mu+}$, $g_R\bar{u}_R\gamma_\mu d_R W_R^{\mu+}$, $\xi W_{L\mu}^+ W_R^{-\mu}$ and masses $m_u\bar{u}_L u_R$, $m_d\bar{d}_L d_R$ there is only one complex phase one can not remove by redefinition. Therefore, if any one of them is zero, the CP violation will disappear to this approximation. That means all the CP violating quantities originating from this mechanism have to be proportional to $g_L g_R \xi m_u m_d$. (This argument has to be modified in some cases as typified by Example 1 to be discussed in the Section E.)

Unfortunately, we can not really set all the other CP violating phases to zero. This is because the left-right mixing phase alone can not explain the $\epsilon$ parameter of kaon system. This is the consequence of a peculiar fact. According to Eqn( 31), in order to calculate $\epsilon$ we have to calculate the real and imaginary parts of $\Delta s = 2$ kaon mass matrix element, $M_{12}$. Both are due to the so-called left-right box diagrams in Fig. 8.

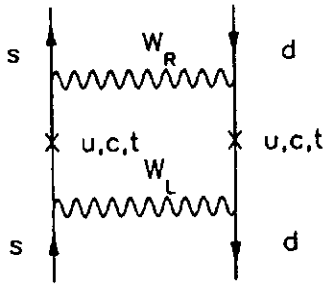


Figure 8: Feynman diagrams contributing to $\Delta m^K$ and $\epsilon$, the left-right box diagrams.

Beall, Bander and Soni[46] discovered that, besides being suppressed by the right-handed scale, the contributions of these diagrams have a factor of 430 enhancement compared to the usual left-left box diagram. This is enhancement is a result of an extra combinatorial factor (of 2), a large $\gamma$ matrix algebra factor (of 4), a reduced GIM cancellation effect(from power to logarithmic suppression), and an enhanced hadronic matrix element. Since the left-left box is well kown to have given roughly the right value for the observed $\Delta m_K$, their calculation implies $430(M_{W_L}/M_{W_R})^2 \leq 1$. This is the main reason that the lower bound on the mass of the right-handed gauge boson is usually taken to be $1.6TeV$.

If the left-right box diagrams contribute to both the real and the CP violating imaginary part then these large enhancement factors are cancelled. One can of course include the effect of left-right mixing in the $W_R$ propagator in the diagram. However in that case not only do the enhancement factors disappear, but the helicity of the operator requires external(light quark) mass insertions, or an additional left-right mixing insertion. In both cases the contribution is too small to explain the observed $\epsilon$. Note however that in those models in which the CP is spontaneously broken, even though the left-right mixing phase is the only phase not suppressed by fermion mass ratios, the induced phases in the right-handed currents is naturally small enough to explain $\epsilon$[19].

The main source of contribution to $\epsilon'$ however can be due to the left-right mixing



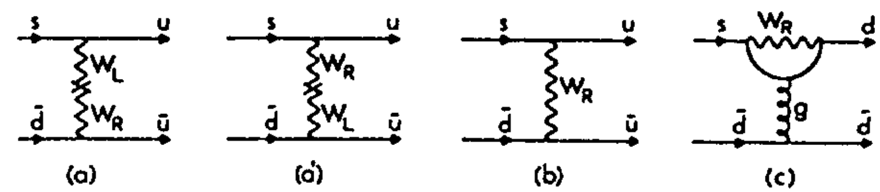


Figure 9: Feynman diagrams contributing $\epsilon'$.

as in Fig. 9.

A recent calculation[22] gives

$$\left| \frac{\epsilon'}{\epsilon} \right| = 550\xi\eta. \tag{47}$$

The recent experimental results imply $\xi\eta \leq 3.6 \times 10^{-6}$. Therefore it is possible that both $\xi$ and $\eta$ are about $10^{-3}$.

The neutron electric dipole moment can arise from the left-right mixing phase alone. The relevant diagram is given in Fig. 10. It was first calculated by Beall and Soni and followed by many groups[21]. To estimate the diagram, one notes that left-right mixing in the gauge boson propagator implies that one has to make a mass insertion in the internal fermion line. This argument confirms the necessity of the combination $g_L g_R \xi m_u m_d$ for CP violation mentioned before. One may wonder why we do not get an external mass insertion factor as expected. In fact that can be understood very simply by following the same argument in Example 1 of Section E. For light internal fermions, the diagrams with photon attached to the fermion line will get an logarithmic infrared divergence as a result of the internal mass insertion. Therefore these diagrams will dominate over the others. One can get the estimate

$$\frac{d_q}{e} \simeq Q_{q'} \frac{g_L g_R}{(4\pi)^2} \frac{m_{q'}}{M_L^2} 2\xi f\left(\frac{m_{q'}}{M_L}\right) \tag{48}$$

where $f(x)$ is a function of order one if $x \sim 1$; for small $x$, $f(x)$ is expected to be $\simeq$ constant $\times \ln x$ as a result of the infrared property. Explicit loop calculations

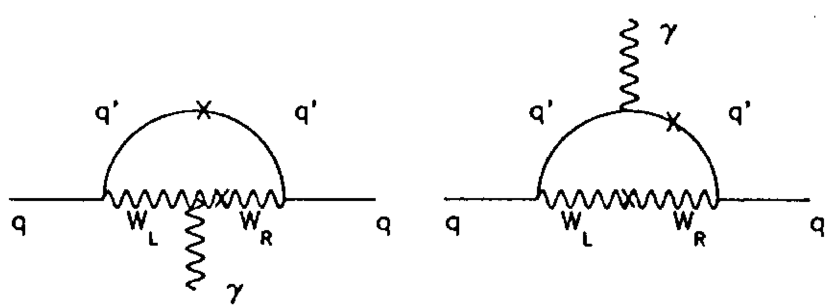


Figure 10: Feynman diagrams contributing to the electric dipole moment of the quarks in the left-right models.

confirm this expectation[21]. A recent calculation gives the numerical result $D_N = 1.8 \times 10^{-18} \xi\eta e - cm$ which relates to

$$| D_N |= 3.6 \times 10^{-24} | \frac{\epsilon'}{\epsilon} | \tag{49}$$

This gives rise to $D_N$ close to the recent experimental bound if the value of $\frac{\epsilon'}{\epsilon}$ is as large the experimental limit.

For the one generation case, one can use a phase definition such that the Yukawa couplings are made real. Then the charged current interaction can be written as (taking the third generation for example)

$$\mathcal{L}_C = \bar{t}\gamma_\mu(a_i + b_i\gamma_5)bW_i^{\mu+} + \bar{t}(c_i + d_i\gamma_5)bG_i^+ + h.\ c. \tag{50}$$

where

$$a_1 = (g_L \cos\xi + g_R e^{i\eta} \sin\xi)/2\sqrt{2}, \quad b_1 = (-g_L \cos\xi + g_R e^{i\eta} \sin\xi)/2\sqrt{2}, \tag{51}$$

$$a_2 = (-g_L e^{-i\eta} \sin\xi + g_R \cos\xi)/2\sqrt{2}, \quad b_2 = (g_L e^{-i\eta} \sin\xi + g_R \cos\xi)/2\sqrt{2}, \tag{52}$$

and $c_i = a_i(m_t - m_b)/M_{W_i}$, $d_i = b_i(m_t + m_b)/M_{W_i}$. These explicit formulae will be useful for calculations in Part II later.



# E. Supersymmetric theories

Supersymmetric models typically have many sources of CP violation[23, 24, 25, 26]. The detailed interplay between different mechanisms is very model dependent. A detailed discussion of any of the complicated models won't be very fruitful or even general. We shall here simply settle for an illustration of many of the popular mechanisms that have been used in the literature.

*Example* 1. CP violation in supersymmetric QCD.

The mechanism will contribute to the electric dipole moment(EDM) of the neutron, $D_n$, the EDM of quarks, $D_q$,the chromo-EDM of quarks, $D_q^c$, and the chromo-EDM of gluons, $D_G^c$. The Lagrangian can be written as

$$\begin{aligned} -\mathcal{L}_W &= \{g_s(\bar{q}_L\gamma^\mu\lambda^a q_L + \bar{q}_R\gamma^\mu\lambda^a q_R)g_\mu^a\} \\ &+ \{g_s'(\bar{\tilde{q}}_L\lambda^a\bar{(q^c)}_R\tilde{g}_L^a + \bar{\tilde{q}}_R\lambda^a\bar{q}_R\tilde{g}_L^a) + h.\ c.\ \} \\ &+ \{m_L^2\bar{\tilde{q}}_L\tilde{q}_L + m_R^2\bar{\tilde{q}}_R\tilde{q}_R + m_{LR}^2\bar{\tilde{q}}_L\tilde{q}_R + M_G\bar{\tilde{g}}_L^a\tilde{g}_L^a + h.\ c.\ \} \\ &+ m_q\bar{q}_L q_R + h.\ c. \end{aligned} \tag{53}$$

where $q$ and $g_\mu^a$ are quark and gluon fields. The fields with a tilde are the superpartners of these fields. The terms in the two brackets are part of the supersymmetric QCD Lagrangian if $g_s = g_s'$. The terms in the second bracket are terms which break supersymmetry softly which is the standard way of implementing supersymmetry breaking. The last term is of course the usual quark mass term.

The complex parameters in this theory are $g_s'$, $m_{LR}$, $M_G$ and $m_q$. They are the potential sources of CP violation. Due to supersymmetry, $g_s'$ is in general real and it is convenient to choose only phase conventions such that this is the case. The phase of $m_{LR}$ can be absorbed into the redefinition of the phase of $\tilde{q}_R$ and $\bar{q}_R$ without affecting the supersymmetric terms. The phase of $M_G$ can be absorbed into $\tilde{g}_L^a, q_R$ and $(q^c)_R$ without affecting supersymmetric terms and $m_{LR}$. If we pick the phase convention such that $m_{LR}$ and $M_G$ are real then the quark mass $m_q$ is in general complex and there is no redefinition one can do to remove the phase without moving the phase to the other terms. One can also use the same argument to show that if any two of the three complex massive parameters are chosen to be real by proper definition of complex fields then the remaining one will be complex and its phase

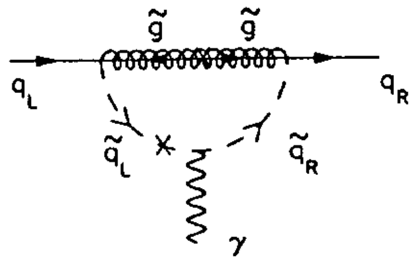


Figure 11: Feynman diagrams contributing to the electric dipole moment of the quark in the supersymmetric QCD models.

will be a genuine CP violating phase. Therefore we expect quite generally that the CP violating quantities will be proportional to $(g_s')^2 m_{LR}^2 M_G m_q$. This is the power of recognizing where the CP violation resides.

Note that if the soft breaking term is induced from some simple high energy supergravity theory there in general may be relations between these soft breaking parameters[27]. For example, $m_{LR}^2$ in most of the cases is proportional to $m_{3/2} m_q$. In some case the proportional constant may even be real. However that does not interfere with our CP argument as long as they are both nonzero and have inconsistent phase with $M_G$.

As an application, one can use this theory to calculate the electric dipole moment of the quark $q$. The diagram is shown in Fig. 11. To contribute to $D_q$, the incoming quark and outgoing quark have to have different helicity. As a result all the mass insertions indicated by a cross in the diagram are forced upon us. One can easily estimate the diagram to be proportional to (two loop factor)$\cdot(g_s')^2 m_{LR}^2 M_G$. Checking with our estimate, since the $m_q$ factor is not present one may wonder whether we really have the CP violating effect in the diagram or not. The answer is that in the limit that the quark mass is zero, we will still get the same electric dipole moment from the diagram; however, in this case the electric dipole moment is not a CP violating operator at all. For a massless quark, one has chiral symmetry which can



be used to rotate an electric dipole moment into a magnetic dipole moment which is always CP conserving. In other words, due to the enlarged (chiral) symmetry, one can defined CP symmetry for both electric and magnetic dipole moments. Even though the diagram does induce a electric dipole moment operator without explicit $m_q$ dependence, we can not claim that the result implies CP violation unless we know the associated quark mass is nonzero. Therefore, there is no inconsistency with our earlier requirement on CP violation. We have learned that when the induced CP violating operator invloves chiral quarks in the external line it may not be necessary to have $m_q$ factor in the diagram. However, the result can be interpreted as CP violation only if the quark mass is nonzero. This is a general feature of CP violation which is by no mean particular to this example. In fact we have already encountered this situation before when we discussed the electric dipole moment of quarks in the left-right models. One has to keep this in mind when appying the general argument about the existence of CP vioalting effect in a diagram.

*Example* 2. CP violation in supersymmetric QED.
The Lagrangian in this case is almost the same as in the previous case except that the color factors can be ignored, the quark and gluon fields can be replaced by electron and photon fields and similarly for their superpartners. Again, for CP violation to be in a process, the result of the calculation has to be proportional to $(e')^2 m_{LR}^2 M_\gamma m_e$ where $e'$ is the coupling constant for superpartner of the usual gauge interactions. As an example, one can calculate the electric dipole moment of the electron in this model. The diagram is shown in Fig. 12. As in the previous example, one finds that the result is proportional to $(e')^2 m_{LR}^2 M_\gamma$. Again, the electron mass does not have to be in the formula for the same reason that quark mass did not have to be in Fig. 11.

*Example* 3. CP violation in supersymmetric $SU(2)_L \times U(1)_Y$ gauge theory.
Starting with the usual $SU(2)_L \times U(1)_Y$ gauge interaction one can supersymmetrize it and then add soft breaking terms as before. The result is the new interactions

$$\begin{aligned} -\mathcal{L}_W &= e(\bar{\tilde{w}}_L^+ \gamma^\mu \tilde{\gamma}_L + \bar{\tilde{\gamma}}_L \gamma^\mu \tilde{w}_L^-) W_\mu^+ \\ &\quad + M_\gamma \tilde{\gamma}_L \tilde{\gamma}_L + m_w \bar{\tilde{w}}_L^+ (\tilde{w}_L^-)^c + h.\ c. \end{aligned} \tag{54}$$

Therefore there are three Weyl fields in the theory, $\tilde{\gamma}_L, \tilde{w}_L^-$ and $\tilde{w}_L^+$. They are superpartners of the photon, $W_\mu^+$ and $W_\mu^-$ respectively. All three couplings can be complex

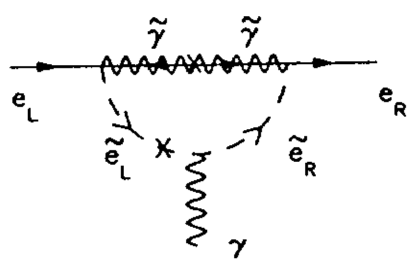


Figure 12: Feynman diagrams contributing to the electric dipole moment of the electron in the supersymmetric QED models.

and potentially CP violating. However, two of them can be absorbed into redefinition of the fields. Therefore we expect any CP violating consequences to be proportional to $\epsilon' M_\gamma m_w$. As an application, we can calculate the electric dipole moment of the W boson. A typical relevant diagram is given in Fig. 13 where all the insertions are shown with crosses which demonstrate how these factors that we predicted are realized.

Another application of this theory is to calculate the chromo electric dipole moment of the gluon. We shall discuss this in Part II.

# Part II. Recent Developments

## A. The Electric Dipole Moment of the Neutron

Recent excitement about neutron electric dipole moment, $D_N$, and CP violation in general was fueled by a sequence of experimental improvements on measurements of CP violating quantities. It started with the measurement of $D_N$ by the Leningrad Group[50]. They observed $D_N = (-14 \pm 6) \times 10^{-26}$ e-cm. Followed by two improved measurements of $\epsilon'/\epsilon$ by the NA31 experiment[51] at CERN and the E731



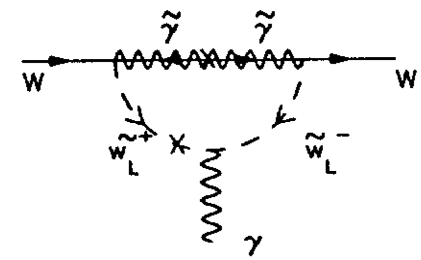


Figure 13: Feynman diagrams contributing to the electric dipole moment of the W boson in the supersymmetric $SU(2)_L \times U(1)_Y$ models.

experiment[43] at Fermilab. Their measurement can be summarized as:

$$\epsilon'/\epsilon = \begin{cases} (33 \pm 11) \times 10^{-4} & (NA31) \\ (-4 \pm 15) \times 10^{-4} & (E731) \end{cases} \tag{55}$$

Then the measurement of $D_N$ was improved by the Grenoble Group[52] with $D_N = (-3 \pm 5) \times 10^{-26}$ e-cm. These experiments showed inconsistencies at a certain confidence level and all of them are improving their results by further measurements. On another front, a recent atomic physics measurement[53] of the electric dipole moment of the electron gave $(-1.5 \pm 5.5 \pm 1.5) \times 10^{-26}$. Most recently, a new measurement[54] of electric dipole moment of electron using $^{205}Tl$ atom obtained $D_e = (-2.7 \pm 8.3) \times 10^{-27}$ based on the atomic electric dipole moment of $D_a = (1.6 \pm 5.0) \times 10^{-24}$. This amounts to about one order of magnitude improvement over the previous result. For later comparison we shall conservatively summarize these experiments as $|D_N| \leq 8 \times 10^{-26}$ e-cm; and an upper bound on $D_e$ of about $10^{-26}$. All these experimental activities rekindled further theoretical investigations into these quantities. They are complemented by many inspiring ideas in the theoretical front which we shall review here. Some reviews which contain older references can be found in Ref.[47]. A shorter review of recent progress can be found in Ref.[15, 48, 49].

We shall start with the discovery by Weinberg[55] of another important mechanism for $D_N$. He showed that there is an unique gauge invariant, $P$-odd, and $T$-odd operator of dimension 6, $O_G$, involving solely the gluon field strength. It can be written as

$$O_G(\mu) = -\frac{1}{3} f^{abc} g_{\alpha\beta} \tilde{G}^a_{\mu\nu} G^{b\mu\alpha} G^{c\nu\beta} , \tag{56}$$

where $\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho} G^{a\lambda\rho}$, $\mu$ is the renormalization scale, and the convention is $\epsilon^{0123} = +1$. Another important operator that can mix with $O_G$ is the color-electric dipole moment(CEDM) operator of quark q,

$$O_q(\mu) = \tilde{G}^a_{\mu\nu} \bar{q} \left(\frac{1}{2}\sigma^{\mu\nu}\right) T^a q , \tag{57}$$

where $T^a$ is the generator of color $SU(3)$ in the fundamental representation and $\frac{1}{2}\sigma^{\mu\nu}$ is the spin matrix for the Dirac spinor representation of the Lorentz group. One can identify the operator $O_G$ as the color-electric dipole moment operator of the gluon itself by expressing it as an analogous form:

$$O_G(\mu) = -\frac{1}{12} \tilde{G}^a_{\mu\nu} G^b_{\alpha\beta} \left(S^{\mu\nu}\right)^{\alpha\beta,\gamma\delta} \left(F^a\right)_{bc} G^c_{\gamma\delta}, \tag{58}$$

where $(F^a)_{bc} = i f^{bac}$ is the color matrix for the adjoint representation and $S^{\mu\nu}$ is the spin matrix for the antisymmetric tensor representation of the Lorentz group. This matrix can in fact be represented compactly using Dirac algebra:[56]

$$\left(S^{\mu\nu}\right)^{\alpha\beta,\gamma\delta} = -\frac{i}{64} \mathrm{Tr}\left([\gamma^\alpha,\gamma^\beta][\gamma^\mu,\gamma^\nu][\gamma^\gamma,\gamma^\delta]\right). \tag{59}$$

This matrix does indeed satisfy the algebra of generators of the Lorentz group:

$$\left[S^{\mu\nu}, S^{\lambda\rho}\right] = i\left(g^{\nu\lambda} S^{\mu\rho} + g^{\mu\rho} S^{\nu\lambda} - g^{\nu\rho} S^{\mu\lambda} - g^{\mu\lambda} S^{\nu\rho}\right) . \tag{60}$$

Then we have $\delta G^a_{\alpha\beta} = \frac{1}{2}(S^{\mu\nu})_{\alpha\beta,\gamma\delta}\omega_{\mu\nu} G^{a\gamma\delta}$, under an infinitesimal Lorentz transformation. Thus the operator $O_G$ can be identified as the color-electric dipole moment of the gluon field strength.

Alternatively one can deduce the color-electric dipole moment of the gluon by starting with the color electric and magnetic fields. There exist only two $P$-odd, $T$-odd, dimension 6, gauge invariant operators involving color-electric and magnetic fields: $f^{abc}(\vec{E}^a \times \vec{E}^b) \cdot \vec{E}^c$ and $f^{abc}(\vec{B}^a \times \vec{B}^b) \cdot \vec{E}^c$. These operators have rotational invariance but not Lorentz invariance. The Lorentz boost will mix the color-electric and color-magnetic fields. Therefore, by imposing Lorentz invariance, one can fix the relative coefficients of the two operators and deduce [49, 56]:

$$O_G = f^{abc}[\frac{1}{3}(\vec{E}^a \times \vec{E}^b) - (\vec{B}^a \times \vec{B}^b)] \cdot \vec{E}^c. \tag{61}$$



It is natural to identify the combination in the bracket as the color electric dipole moment of the gluon.

Knowing the existence of $O_G$ leaves us with three tasks. The first one is to calculate its coefficient, $C$, in a specific model of CP violation and determine the scale at which the operator is induced. The scale is deduced by analyzing the leading contributions in the integration over loop momenta.

The second task is to evolve the operator at whatever scale it is induced to the low energy scale where its physical effect is measured. The leading evolutionary effect is of course due to the QCD correction. Such a correction can be evaluated using the renormalization group technique which is designed to sum over leading QCD logarithmic corrections to arbitrary loop level.

The third task is to evaluate its contribution to the $D_N$ by calculating the hadronic matrix element $\langle$ neutron + photon $|O_G|$ neutron $\rangle$.

To make matters more interesting, all three tasks are intricately coupled. To evaluate $C$ in a specific model one should incorporate QCD renormalization group (R.G.) correction as much as possible. This involves first integrating out the heavy particles one by one in the model and investigating at which stage the operator $O_G$ is induced as a local operator. In some cases, other operators such as $O_q$ are induced first and $O_G$ is induced later only through the matching conditions at the threshold where another set of fields such as the quark q is integrated out. Then the operator is evolved to the hadronic scale taking into account its possible mixing with all the existing operators at any particular scale using R.G. technique.

On the other hand, the result of the R.G. calculation is very sensitive to the choice of the hadronic scale. The proper hadronic scale to choose is presumably determined by the scale at which we think the hadronic matrix element can be evaluated with confidence. One may think that the neutron mass should be the natural scale to use for the low energy end of the R.G. evolution. However, unfortunately, one is limited by our inability to evaluate the matrix element especially for the operators involving only gluons.

The simplest way to estimate the matrix element is to use "naive dimensional analysis"[57]. Manohar and Georgi[57] designed a systematic scheme for doing the dimensional analysis. Every interaction $gO$ is assigned a reduced coupling $(4\pi)^{2-N} M_X^{D-4} g$ where $D = dim(O)$ and the operator $O$ contains $N$ fields. $M_X$ is the chiral symmetry breaking scale $= 2\pi F_\pi = 1190$ MeV. When one evaluates the combined contribution of operators involving quarks and gluons to a hadronic process one simply identifies the reduced coupling of the corresponding hadronic operator with the product of the reduced couplings at the constituent level. For example, the interaction $CO_G$ has the reduced coupling $C(4\pi)^{-1}M_X^2$. The electromagnetic coupling has the reduced coupling $e(4\pi)^{-1}$. Together they can induce the neutron electric dipole moment at the hadronic level. Therefore the neutron electric dipole moment operator, $D_N \bar{N}\sigma_{\mu\nu}\gamma_5 N F^{\mu\nu}$, has the reduced coupling

$$D_N \cdot \frac{M_X}{(4\pi)} = \frac{C}{4\pi} M_X^2 \cdot \frac{e}{4\pi}. \tag{62}$$

$D_N$ can be written as[55]

$$D_N \sim e M_X \zeta_{QCD}(\mu)(g_s(\mu)/4\pi)^{-3} C(g_s(\mu)), \tag{63}$$

where $\mu$ is a hadronic scale, and $\zeta_{QCD}$ is the QCD renormalization factor.

Manohar and Georgi argued that this dimensional estimate of matrix elements should be reliable only for the matrix elements which involve scales near the confinement scale ( $\sim 250$ MeV) which is below the chiral symmetry breaking scale. It is not clear how reliable it is for estimating the matrix element of a purely gluonic operator between neutrons.

Compounding the problem is the fact that near the confinement scale the QCD coupling is known to be strong and the perturbative R.G. analysis is invalid there. In fact below the chiral symmetry breaking phase transition, the QCD coupling constant is supposed to be corrected to give a smaller value as compared to that predicted by the extrapolation of R.G. analysis. In the same way one also expects the anomalous dimensions of various operators, including $O_G$, to be corrected due to the presence of the Goldstone modes. All of this qualitative argument is, however, difficult to quantify. For this reason we shall take the chiral symmetry breaking scale as our low energy end of the R.G. evolution and consider the uncertainty in the choice of the hadronic scale part of the theoretical uncertainty.



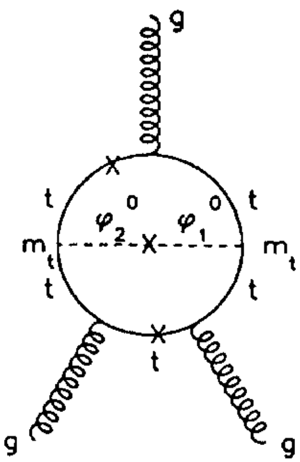


Figure 14: Feynman diagrams contributing to the Weinberg operator $O_g$ in the neutral Higgs models.

To get an idea of how uncertain the estimate of Eqn( 63) is, one can compare it with another recent estimate of this matrix element[58] which obtained a value 30 times smaller than Eqn( 63).

Modulo this uncertainty, the experimental results on $D_N$ may still give strong constraints on models that give rise to appreciable $C$. Weinberg [55] showed that $C$ is induced, for models with $CP$ violating mixing of the physical neutral Higgs bosons, through the two–loop diagrams shown in Fig. 14. The leading contribution is coming from the t-quark in the loop. The diagram in Fig. 14(b) contributes only to the self energy correction of the t quark and therefore does not contribute to the operator $O_G$.

For a general neutral Higgs boson exchange interaction, it is easy to see that the CP violation requires the helicities of the two Yukawa couplings to be correlated. The general Yukawa coupling for neutral Higgs can be written as

$$\mathcal{L}_Y = a\bar{q}_L q_R + a^*\bar{q}_R q_L. \tag{64}$$

For CP violation, one needs either $a^2$ or $(a^*)^2$ from the two Yukawa vertices in Fig. 14, so that they are correlated in such a way that one is forced to pick up one of the $m_q$ parts of the numerators of the quark propagators between the two consecutive Yukawa couplings. We call this the mass insertion on the fermion line. Note that this is a general feature of any CP violation due to the scalar-pseudoscalar neutral scalar boson exchange. Knowing this basic feature, we can go ahead to estimate the contribution of the diagram. From the above argument, we immediately expect to get four powers of $m_q$, two from the Yukawa couplings and two from the fermion mass insertions.

However, this does not yet reflect the behavior of the loop contribution when the quark mass is taken to be small. The reason is that the loop integral may be infrared divergent when the quark masses are taken to be vanishingly small. Before we show this, we first translate the operator $\mathcal{O}_G$ into momentum space. Up to an additive term which vanishes on shell, the tensor, $\mathcal{T}$, of the three gluon vertex corresponding to the operator $\mathcal{O}$ in momentum space can be written as[36]

$$\mathcal{T} = -i\frac{1}{16} f^{abc}\mathrm{Tr}\left([\not k_1,\gamma^\mu][\not k_2,\gamma^\nu][\not k_3,\gamma^\lambda]\gamma_5\right). \tag{65}$$

Therefore to contribute to $C$ one needs to pick up three powers of external momenta. In Fig. 14(a), there are five quark propagators. Among their numerators, two will pick up the quark mass pieces. The other three will have to pick up the external momenta part of their $\not p_i$ pieces. After all the helicity and external momenta are accounted for, one can set the external momenta to zero in the remaining integral. Now it is clear that in the loop with three fermion propagators the loop integration will have quadratic infrared divergence when the quark mass is set to zero. The loop with two fermion propagators will have logarithmic infrared divergence in the same limit. That means one should expect a factor of $(m_q)^{-2}\ln(m_q)$ from the loop integral. With this one can easily write down an estimate of the result

$$C = \eta^{NH}_{QCD}\left(\frac{g_s^3 g^2}{4(4\pi)^4}\right)\left(\frac{1}{M_W^2}\right)\mathrm{Im}Z_2\, h_{NH}(\frac{m_t}{M_H}) \tag{66}$$

where $\eta^{NH}_{QCD}$ is the QCD correction factor to be determined later. The second factor is the typical contribution of any two loop diagram. $g$ and $g_s$ are $SU(2)_L$ and $SU(3)$ coupling constants respectively. The $(1/M_W)^2$ factor follows from the dimension argument and the fact that the Yukawa couplings are proportional to $(1/M_W)$. The $\mathrm{Im}Z_2$ parametrizes the CP violation in the theory as explained previously Section I.B. $h_{NH}(\sigma)$ is a function of order one when the variable $\sigma$ is of order one. For small $\sigma$, we expect $h_{NH}(\sigma)$ to have like (constant)$\times(\sigma)^2\ln(\sigma)^2$. The explicit two loop calculation gives

$$h_{NH}(\sigma) = \frac{\sigma^4}{2}\int_0^1 dx\int_0^1 dy\frac{y^3x^3(1-x)}{[x(1-xy)\sigma^2+(1-x)(1-y)]^2}. \tag{67}$$



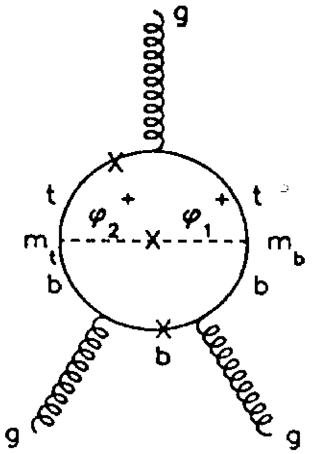


Figure 15: Feynman diagrams contributing to the Weinberg operator $\mathcal{O}_g$ in the charged Higgs models.

In the small $m_t/M_H$ limit,

$$h_{NH}(m_t/M_H) \to \frac{-1}{2}\frac{m_t^2}{M_H^2}\ln\left(\frac{m_t^2}{M_H^2}\right). \tag{68}$$

This confirms our estimate modulo a factor of order one.

Dicus [39] calculated the contribution for models with $CP$ violating charged Higgs boson mixing[6, 8, 9, 10]. The relevant diagrams are given in Fig. 15. From the general helicity argument in the previous paragraph, one expects to have two factors of $(m_b m_t)$ with a factor from the Yukawa couplings and another one from the fermion mass insertions. It is clear that the contribution of Fig. 15(a) is going to be much larger than those of Fig. 15(b) because the loop with the b-quark has more severe infrared divergence. Therefore we can just estimate the contribution of Fig. 15(a). Our estimate can again be given by Eqn( 66) except that the h function is now a function of two variables, $h_{CH}(\sigma_b,\sigma_t)$ where $\sigma_q = m_q/M_H$. The mass $M_H$ in this case is the charged Higgs boson mass. The QCD correction factor will be different as we shall discuss later. For small $\sigma_q$ we expect $h$ to be (constant)$\times(m_t/M_H)^2\ln(m_t/M_H)^2$ where the constant is of order one as with any other "constant" in this paper. We do not expect a suppression factor due to $m_b$ because of the infrared divergnece mentioned earlier. The explicit two loop calculation gives

$$h_{CH}(\sigma_b,\sigma_t) = \frac{\sigma_b^2\sigma_t^2}{2}\int_0^1 dx\int_0^1 dy\frac{y^3x^3(1-x)}{[x(1-x)y\sigma_b^2+x(1-y)\sigma_t^2+(1-x)(1-y)]^2}$$

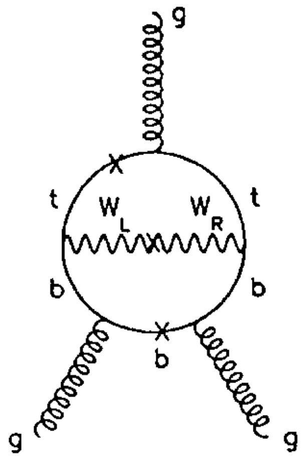


Figure 16: Feynman diagrams contributing to the Weinberg operator $O_g$ in the left-right models.

$$+(\sigma_b \leftrightarrow \sigma_t). \tag{69}$$

For the small $\sigma_q$ limit, we can easily check our estimate and determine the constant to be -1/2 just like the neutral Higgs boson case.

In ref.[60] it was pointed out that the Weinberg mechanism may also provide an appreciable contribution to the $D_N$ for models with $CP$ violating left-right mixing[16, 18, 20]. The relevant diagrams are given in Fig. 16. Following the same arguments as in previous paragraphs, we expect the Fig. 16(b) to be negligible compared to Fig. 16(a). Note however that in this case all the couplings are gauge couplings instead of Yukawa couplings as in the previous two examples. An immediate consequence of this is that the quadratic $(m_b)^2$ infrared singularity is not completely cancelled by the masses from the helicity argument. As a result, there is a net $(1/m_b)$ factor which means the lighter quark mass actually provides an enhancement effect instead of suppression as one may naively expect.

The actual two loop calculation gives

$$C = -\frac{g_s^3}{(4\pi)^4}\sum_{i=1}^{2}\frac{\mathrm{Im}(a_i b_i^*)}{M_{W_i}^2}[h_W(\sigma_{bi},\sigma_{ti}) + h_G(\sigma_{bi},\sigma_{ti})]. \tag{70}$$

where the couplings $a_i, b_i$ are given in Section I.D., and $\sigma_{qi} = m_q/M_{W_i}$. The functions



$h_W$ and $h_G$ are given by

$$h_W(\sigma_{bi},\sigma_{ti}) = 4\sigma_{bi}\sigma_{ti}h(\sigma_{bi},\sigma_{ti}), \tag{71}$$

and

$$h_G(\sigma_{bi},\sigma_{ti}) = (\sigma_{ti}^2 - \sigma_{bi}^2)\sigma_{bi}\sigma_{ti}h'(\sigma_{bi},\sigma_{ti}), \tag{72}$$

with

$$\begin{aligned} h(\sigma_{bi},\sigma_{ti}) &= \int_0^1 dx\int_0^1 dy\frac{y^3x^2(1-x)^2}{[x(1-x)y\sigma_{bi}^2 + x(1-y)\sigma_{ti}^2 + (1-x)(1-y)]^2} \\ &\quad -(\sigma_{bi}\leftrightarrow\sigma_{ti}) \end{aligned} \tag{73}$$

and

$$\begin{aligned} h'(\sigma_{bi},\sigma_{ti}) &= \int_0^1 dx\int_0^1 dy\frac{y^3x^3(1-x)}{[x(1-x)y\sigma_{bi}^2 + x(1-y)\sigma_{ti}^2 + (1-x)(1-y)]^2} \\ &\quad +(\sigma_{bi}\leftrightarrow\sigma_{ti}). \end{aligned} \tag{74}$$

Note that to obtain $CP$ violation in Fig. 16(a) one has to pick the left-handed vector coupling at one end of the $W_i$ propagator and the right-handed vector coupling at the other. As a result one has to make one mass insertion on each of the two quark lines which explains the $m_t m_b$ factor in $h_W$. To have $CP$ violation for diagrams in which $W_i$ is replaced by the Goldstone bosons, one has to use scalar vertices of the same chirality at both ends of the unphysical Higgs propagator. As a result one picks up either a factor of $m_t^2 e^{-i\eta}$ or a factor of $m_b^2 e^{i\eta}$ from the vertices and a mass insertion on each of the two quark lines. This explains the factor of $m_b m_t(m_t^2 - m_b^2)$ in $h_G$.

For small $\sigma_{bi}$, but arbitrary $\sigma_{ti}$, we have

$$h(\sigma_{bi},\sigma_{ti}) \sim \frac{1}{\sigma_{bi}^2(1-\sigma_{ti}^2)^3}(\sigma_{ti}^2\ln\sigma_{ti}^2 + \frac{1}{2} - \frac{1}{2}\sigma_{ti}^4) \tag{75}$$

$$h'(\sigma_{bi},\sigma_{ti}) \sim \frac{1}{\sigma_{bi}^2(1-\sigma_{ti}^2)^3}(-\ln\sigma_{ti}^2 - \frac{3}{2} + 2\sigma_{ti}^2 - \frac{1}{2}\sigma_{ti}^4) \tag{76}$$

and thus

$$h_W + h_G \sim \frac{\sigma_{ti}}{\sigma_{bi}(1-\sigma_{ti}^2)^3}(2 + 3\sigma_{ti}^2\ln\sigma_{ti}^2 - \frac{3}{2}\sigma_{ti}^2 - \frac{1}{2}\sigma_{ti}^6). \tag{77}$$

Using Weinberg's estimation[65] of the matrix element, $D_n$ is given by (for simplicity, we assume $g_L = g_R = g_2$ and $M_{W_2} \gg M_{W_1} = M_W$)

$$D_n \sim 1.59\times 10^{-19} f(\frac{m_t}{M_W})\zeta\sin\xi\sin\eta \quad \mathrm{e-cm} \tag{78}$$

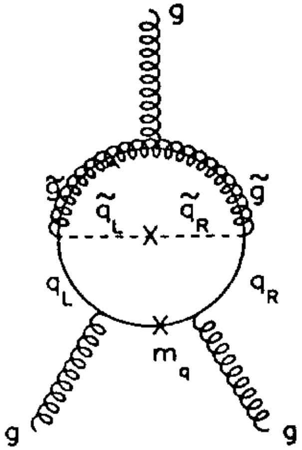


Figure 17: Feynman diagrams contributing to the Weinberg operator $O_g$ in the supersymmetric models.

where $f(x) = (2x + 6x^3 \ln x - 3x^3/2 - x^7/2)/(1-x^2)^3$. For $m_t/M_W \geq 1$, the function $f(m_t/M_W)$ is about $(-1/2)(m_t/M_W)$. In order to compare with the contribution of the one loop electric dipole moments of the constituent quarks to the electric dipole moment of neutron calculated by Beall and Soni[21]

$$D_n(\text{Beall} - \text{Soni}) \sim 5.3 \times 10^{-21} \sin\xi \sin\eta \quad e-\text{cm}, \tag{79}$$

we need to know the QCD evolution factor $\zeta$ which we will be discussed shortly.

For supersymmetric models, there can be many sources of $O_G$. The most important one is probably the supersymmetric QCD mechanism mentioned Section I.E. As mentioned there, the CP violating effect has to be proportional $(g_s')^2 m_{LR}^2 M_G m_q$. This is indeed demonstrated clearly by crosses in the most important two loop diagram, shown in Fig. 17. Note that in this case heavy as well as light quarks can contribute to the loop. As a result of the infrared property of the quark loop we get an $(1/m_q)^2$ factor from the loop which combined with the CP factor mentioned above result in a net $(1/m_q)$ factor as in the case of left-right models. This diagram has been calculated by many groups[23, 24, 25]. It certainly confirms our expectation and estimate. We shall not get into detail here because the basic feature is similar to the other models that we have discussed.

The fermion mass singularity in both the charged Higgs and the left-right models



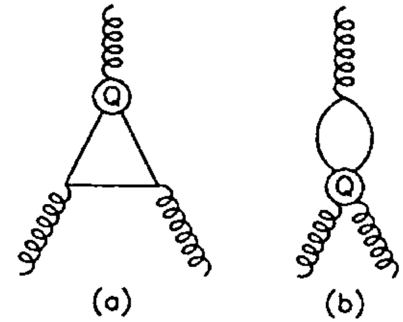


Figure 18: Diagrams that determine the shift in the coefficient $C_G(\mu)$ at the threshold for a heavy quark $Q$.

is of course a signal that the corresponding fermion loop should not be treated as a local operator at the scale above the fermion mass scale which in this case is the b quark mass. The proper way to treat this problem is to integrate out the t quark and the W boson first which induces a color electric dipole moment operator($O_b$) for the b quark as shown in Fig. 18(a).

This operator is then QCD corrected using R.G. technique down to the b-quark scale. The $O_G$ operator is subsequently induced after the b quark is integrated out as shown in Fig. 18(b). That is, for this class of models, in order to take the leading logarithmic QCD correction into account one should break up the two loop calculations discussed earlier and keep only the leading term in the scale ratio $m_b/M_W$. Since the leading log. correction is numerically more significant then the higher order term in $m_b/M_W$, it is a worthwhile trade off. One can of course include both leading log. and higher order terms in $m_b/M_W$; however it will take a lot more hard work on the threshold effect and probably will be numerically insignificant.

To calculate the QCD effect, one first has to include all the operators that can mix with $O_G$ at some level in the analysis. That means one should at least include the operator $O_q$ in the analysis because in some models the quark operator $O_q$ can already be induced at the one-loop level.[61, 62, 24, 25] Up to a total derivative term, $O_G$ and $O_q$ are the only gauge invariant $CP$-violating operators with dimension $\leq 6$

that involve the gluon field strength. Thus, the effective Hamiltonian for the neutron electric dipole moment will contain the following terms

$$\mathcal{H}_{eff} = C_G(\mu)O_G(\mu) + \sum_q C_q(\mu)O_q(\mu) \tag{80}$$

where $C_G(\mu)$ and $C_q(\mu)$ are the Wilson coefficient functions that depend on the model of $CP$-violation. Since $\mu\frac{d}{d\mu}\mathcal{H}_{eff} = 0$ by definition the $\mu$-dependence of the operators has to balance the $\mu$-dependence of the Wilson coefficient functions. Once the evolution of the operators as a function of $\mu$ is calculated, the $\mu$-dependence of the coefficients is determined.

Between any two relevant scales of the theory when the particle content of the theory remains the same, the scale dependence of the operators is determined by the anomalous dimensions of the operators. The anomalous dimensions are in turn determined by the cut-off dependence of the loop corrections to the effective Hamiltonian. Therefore calculating anomalous dimension at one loop level is the same as analyzing the divergent structure of the one loop corrections. Since the induced operator with divergent coefficients may not always be the same form as the original one, the anomalous dimension is in general a matrix. For example, up to dimension 6 , $O_G$ and $O_q$ are the only two operators that are CP violating and therefore can mix with each other. (Again, we have ignored the dimension four $F_{\mu\nu}\tilde{F}_{\mu\nu}$ term because it is a total derivative. More discussion about this is given in K. Choi's talk). Therefore, the renormalization group equations for these operators are

$$\mu\frac{\partial}{\partial\mu}\int d^4x O_q(\mu) = \frac{\alpha_s(\mu)}{4\pi}\gamma_{qq}\int d^4x O_q(\mu), \tag{81}$$

and,

$$\mu\frac{\partial}{\partial\mu}\int d^4x O_G(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left(\gamma_{GG}\int d^4x O_G(\mu) + \gamma_{Gq}\sum_q m_q(\mu)\int d^4x O_q(\mu)\right). \tag{82}$$

The anomalous dimension for $O_q$, is given by[63, 64] $\gamma_{qq} = \frac{23}{3}C_A - 10C_F - \frac{2}{3}N_f$; while for $O_G$ it is given by[64] $\gamma_{GG} = -C_A - 2N_f$.

The equations imply the operator $O_G$ can induce the operator $O_q$, but not vise versa. This is expected because $O_G$ is of dimension 6 and $O_q$ only of dimension 5. This operator mixing is controlled by the anomalous dimension $\gamma_{Gq}$ which is given[64] by $2C_A$. For $SU(3)$, $C_A = 3$ and $C_F = \frac{4}{3}$.



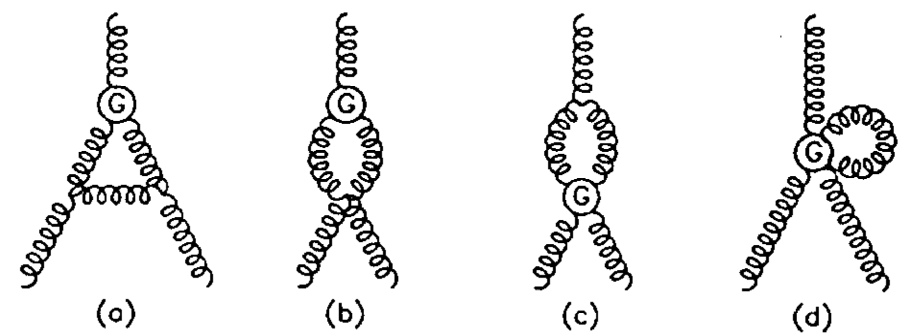


Figure 19: Diagrams that determine the anomalous dimension coefficient $\gamma_{GG}$. The circle with $G$ inside represents the gluonic $CP$-violating operator $O_G$.

In some models a different basis of operators may be more useful for the effective Hamiltonian in (5), such as $O_1(\mu) = g_s(\mu)^3 O_G(\mu)$ and $O_2(\mu) = g_s(\mu)m_q(\mu)O_q(\mu)$, the anomalous dimension matrix is then changed. The diagonal coefficients analogous to $\gamma_{GG}$ and $\gamma_{qq}$ are $\gamma_{11} = \gamma_{GG} - 3\beta = -12C_A$ and $\gamma_{22} = \gamma_{qq} - \beta + \gamma_m = 4C_A - 16C_F$. Here, $\beta = (11C_A - 2N_f)/3$ is the $QCD$ beta-function and $\gamma_m = -6C_F$ is the anomalous dimension of the quark mass operator.

Among these anomalous dimensions, $\gamma_{GG}$ is the most complicated one to calculate because of the complexity of the gluon self-coupling vertex. The diagrams which need to be calculated for this purpose are given in Fig. 19[64]. Since the $O_G$ operator contains terms which have from 3 gluons to 6 gluons, the simplest way is to select a particular term that one wishes to induce and assume that the gauge invariance will take care of the rest. Alternatively, one can avoid the gauge ambiguity by considering the effect of the operator on a scattering process involving three gluons. In ref.[65], the background field method is used in which all the external fields are considered to be background fields. For constant background fields, only the dimension 6 piece, which does not contain any momentum in the vertex, is induced. In this method, the final operator structure is simpler but there are more diagrams to calculate. In ref.[64], a more conventional method is used and the operator structure of the dimension 3 piece is used to identify the operator. They detected the wrong sign in ref.[65]. The issue of gauge dependence has been reexamined[66].

After the dust settled, it was pointed out[58] that all these anomalous dimensions were calculated before by Morozov[67]. He calculated the anomalous dimensions matrix for almost all the QCD operators with dimension $\leq 8$ excluding only those with four fermion couplings. Later, we will discuss some of Morozov's result a little more.

If one is only interested in the low energy effect of the operator $\mathcal{O}_G$, the mixing of the operators $\mathcal{O}_G$ and $\mathcal{O}_q$ is actually a higher order effect and therefore can be ignored[56, 49]. This will be justified shortly. Ignoring this mixing, the solution to the renormalization group equations for the Wilson coefficient functions is then

$$C_G(\mu) = \left(\frac{g_s(\mu)}{g_s(M)}\right)^{\gamma_{GG}/\beta} C_G(M) \quad , \quad C_q(\mu) = \left(\frac{g_s(\mu)}{g_s(M)}\right)^{\gamma_{qq}/\beta} C_q(M). \tag{83}$$

To solve this equation, one needs the initial conditions on the $C$ coefficients in $\mathcal{H}_{eff}$. At the highest scale in the theory, $\mathcal{H}_{eff}$ is the same as the basic Hamiltonian. The initial $C$ coefficients are therefore determined by the input parameter of the theory. As we go down in energy scale, the particles heavier than the relevant scale have to be integrated out. After the heavy particles are integrated out, we obtain a Lagrangian without these particles, however their effects have been summarized in the new effective Hamiltonian. Therefore to make sure their effects are properly taken into account, we should compare the physical effects of the effective Hamiltonian above the heavy particle threshold where these particles are present to the effects of the one below the threshold where such particles no longerpreside. The initial $C$ coefficients for the evolution below each threshold $M$ are thereby determined.

In neutral Higgs models, $\mathcal{O}_G$ is induced when the t quark and $H^0$ are integrated out at some heavy scale $M$. With the initial conditions $C_q(M) = 0$ and $C_G(M) = \tilde{C}_G(M)(g_s(M)/4\pi)^3$, the $CP$-violating effective Hamiltonian becomes

$$\begin{aligned} \mathcal{H}_{eff} = & \ \tilde{C}_G(M)\left(\tfrac{g_s(M)}{4\pi}\right)^3\left(\tfrac{g_s(M)}{g_s(\mu)}\right)^{39/23}\mathcal{O}_G(\mu) \\ = & \ \tilde{C}_G(M)\left(\tfrac{g_s(M)}{g_s(\mu)}\right)^{108/23}\left(\tfrac{g_s(\mu)}{4\pi}\right)^3\mathcal{O}_G(\mu). \end{aligned} \tag{84}$$

We have assumed 5 flavors between the scales $M$ and $\mu$. Note that the $QCD$ correction to the operator $\mathcal{O}_G$ is a suppression effect. Therefore the enhancement effect discussed



in ref.[55] is actually a suppression factor because the exponent of wrong sign was used there.

To discuss the other models we have to emphasize the importance of the operator $\mathcal{O}_q$. This operator does not affect $\mathcal{O}_G$ through its R.G. evolution, but it has a significant effect on the matching condition while going through the threshold of the quark field. At the threshold of a quark $Q$, the operator $\mathcal{O}_Q(\mu)$ for that quark gives a finite contribution to the coefficient of the gluonic operator $\mathcal{O}_G$. Therefore one must match matrix elements of the operator $\mathcal{O}_G(\mu)$ with $\mathcal{H}_{eff}(m_Q^+) \simeq \int C_G(\mu)\mathcal{O}_G(\mu) + \int C_Q(\mu)\mathcal{O}_Q(\mu)$ just above the threshold $\mu = m_Q^+$ with the corresponding matrix elements using $\mathcal{H}_{eff}(m_Q^-) \simeq \int C_G(\mu)\mathcal{O}_G(\mu)$ just below $m_Q^-$, the shift is given by[61, 64]

$$C_G(m_Q^-) = C_G(m_Q^+) + C_Q(M)\left(\frac{g_s(m_Q)}{g_s(M)}\right)^{\gamma_{qq}/\beta}\frac{1}{8\pi}\frac{\alpha_s(m_Q)}{m_Q}. \tag{85}$$

Thus the contribution to $C_G(\mu)$ at the hadronic scale involves an enhancement at the quark thresholds due to $\mathcal{O}_Q's$, followed by suppressions from the evolution of $\mathcal{O}_G$ between thresholds. This shift can be significant when both $C_Q(M)$ and $C_G(M)$ are nonvanishing at the weak scale. In fact, it is easy to see that this is the case in the supersymmetric model[24].

The matching condition also provides a better way to understand the results of Ref.[59] and [60]. The two loop calculations there can be divided into two steps[61, 62], the first being the generation of the operator $\mathcal{O}_b(m_t)$ from a one-loop diagram at the scale $m_t$. In the second step, the operator $\mathcal{O}_b(m_b^+)$ induces the operator $\mathcal{O}_G(m_b^-)$ by the matching condition. This approach has the advantage of summing up all the leading logarithms of the form $\alpha_s^n\log^n(m_t/m_b)$ and thus improves the direct two-loop calculations in these references.

We now justify our earlier claim that operator mixing can be ignored within the approximation we are working at. We need only consider mixing of $\mathcal{O}_G$ with the heavy quark operators $\mathcal{O}_Q$, because hadronic matrix elements involving light quark operators will be suppressed by the light quark mass. If $C_G(M)$ is nonzero, renormalization group evolution of $\mathcal{O}_G$ down to the scale $m_Q$ will generate a contribution to $C_Q(m_Q^+)$ on the order of $C_G(M)m_Q$. Applying the matching condition Eqn( 85) at the heavy quark threshold, we find that the shift in $C_G$ is of order $C_G(M)\alpha_s(m_Q)$. This shift is suppressed by a power of $\alpha_s(m_Q)$ compared to $C_G(m_Q^+)$, and should not be included unless one also computes the order $\alpha_s$ corrections to the initial conditions and to the diagonal evolution of $\mathcal{O}_G$.

If the $CP$-violation comes from the neutral Higgs boson mixing, from Eqn( 66), the NEDM is estimated to be $D_N \sim 2.0 \times 10^{-21} \zeta^{NH}_{QCD}(\mu) h(m_t/M_H)(\mathrm{Im}\ Z_2)$ e − cm where Im $Z_2$ and the function $h(m_t/M_H)$ has been defined earlier. The $QCD$ evolution factor is given by

$$\zeta^{NH}_{QCD}(\mu) = \left(\frac{g_s(M)}{g_s(m_b)}\right)^{108/23} \left(\frac{g_s(m_b)}{g_s(m_c)}\right)^{108/25} \left(\frac{g_s(m_c)}{g_s(\mu)}\right)^{108/27} \left(\frac{g_s(\mu)}{4\pi}\right)^3 . \tag{86}$$

The lower curve in Fig. 20 shows the NEDM ($d_n/\mathrm{Im}Z_2$) as a function of the hadronic scale.

A figure for $D_N$ can be found in ref.[49]. For $\mu \sim 1\ GeV$, $\zeta^{NH}_{QCD} \sim 3 \times 10^{-4}$, and the NEDM is about $6.0 \times 10^{-25}\mathrm{Im}Z_2$e − cm for $m_t \sim M_H$.

Similar calculations can be performed for the charged Higgs boson case.[59] The $QCD$ evolution factor is given by[61]

$$\zeta^{CH}_{QCD}(\mu) = \left(\frac{g_s(M)}{g_s(m_b)}\right)^{28/23} \left(\frac{g_s(m_b)}{g_s(m_c)}\right)^{108/25} \left(\frac{g_s(m_c)}{g_s(\mu)}\right)^{108/27} \left(\frac{g_s(\mu)}{4\pi}\right)^3 . \tag{87}$$

For $\mu \sim 1GeV$, $\zeta^{CH}_{QCD} \sim 10^{-3}$. From the middle curve of Fig. 20, we see that the NEDM is about $3 \times 10^{-26}$Im $Z_2'$ e − cm for $m_t \sim M_{H^+}$.

In the left-right symmetric model, assuming the right-handed scale is the $TeV$ scale, one has[60] $d_n \sim 1.59 \times 10^{-19} \zeta^{LR}_{QCD}(\mu) f(m_t/M_W) \sin\xi \sin\eta$ e − cm, where $\xi$ and $\eta$ are the left-right mixing angle and the $CP$-violation phase respectively. The function $f(x) = (2x + 6x^3 \ln x - 3x^3/2 - x^7/2)/(1-x^2)^3$, and is of order unity for $1 < m_t/M_W < 5$. The $QCD$ evolution factor is

$$\zeta^{LR}_{QCD}(\mu) = \left(\frac{g_s(M)}{g_s(m_b)}\right)^{4/23} \left(\frac{g_s(m_b)}{g_s(m_c)}\right)^{108/25} \left(\frac{g_s(m_c)}{g_s(\mu)}\right)^{108/27} \left(\frac{g_s(\mu)}{4\pi}\right)^3 , \tag{88}$$

$\zeta^{LR}_{QCD} \sim 1.5 \times 10^{-3}$ at $\mu \sim 1GeV$. It is almost an order of magnitude larger than that of the neutral Higgs boson case. From the upper curve of Fig. 20, we have $d_n \sim 2 \times 10^{-22} \sin\xi \sin\eta$ e − cm, where we have assumed $m_t \sim M_W$. Note that for the minimal model[19], $\sin\xi \sin\eta$ is about $1.5 \times 10^{-6}$.



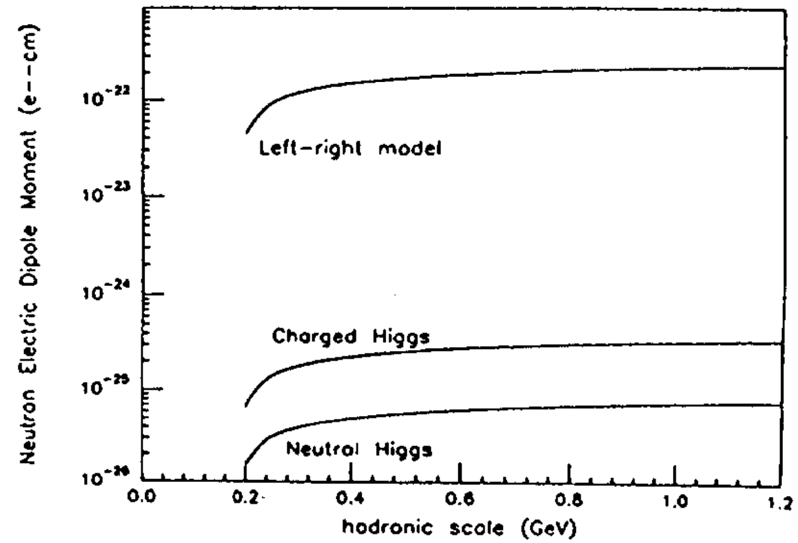


Figure 20: The neutron electric dipole moment ($D_N$), modulo by the corresponding $CP$ violating phases, as a function of the hadronic scale ($\mu$) in various models of $CP$ violation.
Upper curve: $D_N/\sin\xi\sin\eta$ versus $\mu$ in left-right symmetric model; middle curve: $D_N/\mathrm{Im}Z_2'$ versus $\mu$ in charged Higgs boson model; lower curve: $D_N/\mathrm{Im}Z_2$ versus $\mu$ in neutron Higgs boson model. We have set $m_t = M_H = M_{H^+} = M_W$ in these curves.

The first factors of Eqn( 87) and Eqn( 88) come from the evolution of the $b$ quark color-electric dipole moment between $M$ and $m_b$. They have different exponents because the bottom quark mass is absent in the color-electric dipole moment, $C_Q(M)$, of the $b$ quark in the left-right symmetric model as compared to the case of the charged Higgs boson case.[61] It implies that the light quark mass enhancement ($1/m_b$) in the matching condition ( 85) survives for the $D_N$ in the left-right symmetric model.[61]

For supersymmetric models similar analysis can also be done. The general character of this case is not too different from the charged Higgs case[24]. Since the loop with the gluino contains only susy particles which are presumably heavy, it should be integrated out first like the charged Higgs case and induce a chromo-electric dipole moment, $D_q^c$, for the lighter quarks at the supersymmetry breaking scale. The most distinctive feature is that, assuming all the susy particles are of the same scale, the $D_q^c$ of all the quarks are induced at the same time with similar magnitudes. In contrast, in the charged Higgs or the left-right models, only $D_b^c$ for the b quark is induced without the mixing angle suppression.

Before we summarize we wish to mention some interesting problems raised by the calculation of Morozov[67]. As we have seen, the effect of the dimension six gluonic operator is suppressed by the QCD evolution effect. The suppression effect, qualitatively, is not surprising because the operator contains three powers of QCD coupling constants and it gives rise to enhancement effect for the operator and therefore results in suppression in its coefficient. One may wonder whether this is a general feature for other higher dimensional gluonic operators. Morozov's calculation showed that this is not the case. For example, at dimension 8, there are three independent P-, T-violating purely gluonic operators one can write down. They have a 3 by 3 anomalous dimension matrix. The eigenvalues of the matrix are (-86, -41.7, +9.6) using our normalization. The positive eigenvalue indicates that one of the linear combination is actually enhanced by QCD. Given how negative the other eigenvalues are, at low energy, this is almost the only linear combination that will survive. Note that Morozov's result has not been checked in the literature yet. Given the complexity of the calculation it is certainly desirable to have an independent check of the result.

Given this operator with the positive anomalous dimension, one would like to know how important is the low energy effect of this operator. Certainly a higher dimensional operator will be suppressed by the inverse of some higher mass scale. In most of the models, this scale is $M_W$ or higher. Therefore there will be at least a $(m_N/m_W)^2$ suppression factor relative to the dimension 6 operator, $O_G$, where $m_N$ is the neutron mass. It is interesting to see if the QCD enhancement factor can compensate this suppression factor. Using the Higgs models of CP violation as examples, one can identify the two loop diagrams that will contribute to this operator. They are shown in Fig. 21. The diagram in Fig. 21(a) was pointed out by Weinberg before[55]. For neutral Higgs models with the top quark loops, all the diagrams, Fig. 21(a,b,c), give contributions of similar magnitude. For the charged



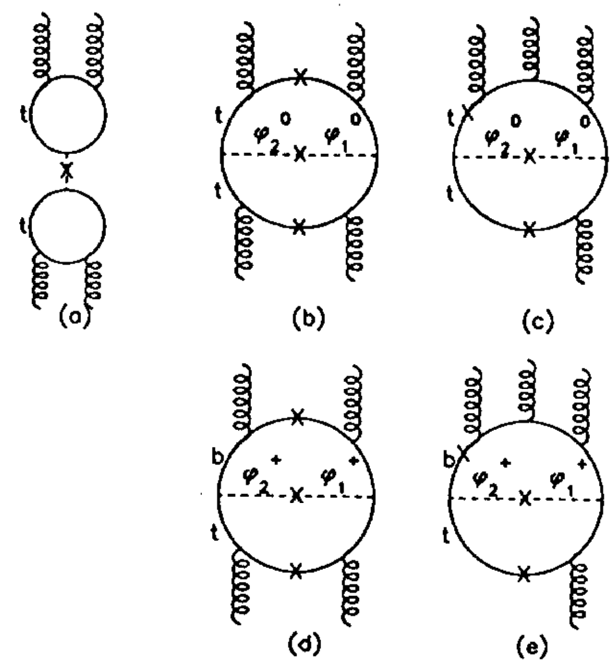


Figure 21: Two loop diagrams that contribute to the dimension 8 purely gluonic operators.

Higgs model, the main contribution is due to the Fig. 21(d,e). This is because the b quark loop is infrared divergent. Therefore one should integrate out the t quark loop first, add the QCD correction and then integrate out the b quark loop. That means, the Fig. 21(e) should dominate over Fig. 21(d) because it has a more severe infrared divergence when $m_b$ is set to zero. Therefore the dimension 8 operator is induced at the $m_b$ scale by the chromo-electric dipole operator of the b quark. Therefore the scale that balances the the dimension should be $m_b$ instead of $M_W$. Of course we also lost the QCD enhancement effect that we sought for earlier between $m_b$ and $M_W$. Without an explicit two loop calculation, we can make a first estimate of its effect. The the two loop graph in Fig. 21(b) for example. It can be estimated to be

$$\mathcal{D}_N \simeq \eta_8^{QCD}\left(\frac{g_s^3 g^2}{4(4\pi)^4}\right)\left(\frac{1}{M_W^2 M_H^2}\right)\mathrm{Im}Z_2\, f_8(\frac{m_t}{M_H}) sin(\theta)\left(\frac{e}{(4\pi)^2}M_X^3\right) \tag{89}$$

where the last bracket is the matrix element of the operator using the naive dimensional estimate; $f_8$ is a function of order one when $m_t \simeq M_H$. The QCD enhancement factor is

$$\eta_8^{QCD} \simeq \left(\frac{g_s(M)}{g_s(\mu)}\right)^{28.8/23} . \tag{90}$$

If $\mu$ is taken to be 250 $MeV$ and $M_H$ taken to be 100 GeV, this contribution can be competitive with that of $\mathcal{O}_G$. A more detail analysis of this is still in progress.

There are also other interesting issues that have been raised in the literature. In ref.[68], it is claimed that the dimension 8 P-, T- violating operators with one photon and three gluon field strengths can give even larger contribution than the dimension 6 operator, $\mathcal{O}_G$. It is also interesting to investigate the importance of these operators in models with the QCD Peccei- Quinn symmetry invented to avoid the strong CP problem[69]. Due to limitations in space and time we shall not explore this further.

Finally, we summarize the contribution of the gluon color electric dipole moment to the neutron electric dipole moment in various models of CP violation. It turns out that this contribution in most of the models in which one can provide a solid prediction of $D_N$ does not represent the dominant contribution. However the mechanism does give rise to nontrivial constraint on the parameters which are not already constrained by CP violation in the kaon sector in a more general class of models. One should however remember that the estimates contain large uncertainties. This is reflected in Fig. 20 that when the scale $\mu$ is less than about 250 $MeV$, all three curves begin to decrease rapidly. We conclude that the mechanism of generating NEDM using the gluon color-electric dipole moment operator does not rule out any reasonable models of $CP$-violation. Nevertheless it remains a powerful mechanism for generation of the neutron electric dipole moment in many models of CP violation.

## B. Electric Dipole Moment of Electron and Quarks

*(1)Barr-Zee mechanism and lelctric dipole moment of elctron*

In the Higgs models of CP violation, at one loop level, the only contribution is through the neutral Higgs boson exchange as shown in Fig. 22. To obtain an electric dipole moment operator the initial and final electrons have to have different helicities. Since in this diagram, the two Yukawa couplings flip helcity twice, we are forced to make



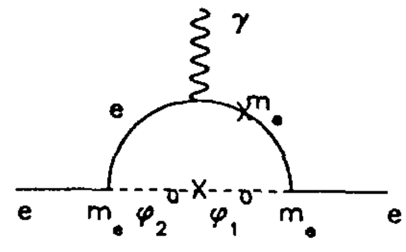


Figure 22: One loop contribution to electric dipole moment of eletron through the neutral Higgs exchange.

one additional mass insertion on the fermion lines. As discussed before in Section II.A., a general character of the neutral Higgs mediated CP violation is that there has to be a mass insertion between the two Yukawa couplings. Therefore the mass insertion required by the helicity of the $D_e$ operator has to be an internal insertion. As a result, the diagram can be estimated to give

$$D_e/e \simeq (g^2/4\pi)(m_e/M_W)^2(m_e/M_H^2)\ln(m_e/M_H)^2, \tag{91}$$

where the logarithmic factor is the anticipated infrared divergence in the loop when the electron mass is set to zero. Two powers of $M_W^{-1}$ originate from the Yukawa couplings. With three powers of $m_e$, the numerical value of this contribution is vanishingly small.

If CP violation is in the charged Higgs sector, it does not even have a one loop contribution. This is because neutrinos are massless and the associated chiral symmetry can be used to rotate away the complex phase of the Yukawa coupling of the lightest charged Higgs boson.

Recently, Barr and Zee[70] pointed out there is a new class of two-loop Feynman diagrams, generically given by Fig. 23 which can lead to a large electric dipole moment (EDM) of the charged leptons or light quarks due to the CP violation in the neutral Higgs propagators.[12, 8] The diagram requires only ony one helicity flip on the

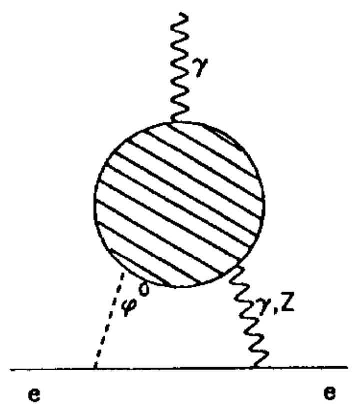


Figure 23: Generic two loop contribution to the electric dipole moment of the electron through an $H\gamma\gamma$ or $HZ\gamma$ vertex.

electron line which is accomplished automatically by the Yukawa coupling. Therefore, generically, the diagram can be estimated to be $D_e/e \simeq (g^2/4\pi)^2(m_e/M_W^2)\cdot f$ where $f$ is a function of order one when all the heavy particles in the loop are of about the same masses.

For the charged Higgs exchange models of CP violation[6, 10, 8, 9], similar two loop diagrams also give important contributions. This case is not investigated in detail in the literature yet[71]. Therefore, we will not discuss this case very much here. We shall assume that neutral Higgs exchange is the sole source of CP violation. Of course, if a model with only two doublets, the charged Higgs sector will be automatically CP conserving.

The two loop diagrams that contribute the $D_e$ can be classified into a few classes. The first one involves a one loop subdiagram through which an effective $H\gamma\gamma$ or $HZ\gamma$ vertices are induced as those in Fig. 24. The first loop in any of these two-loop diagrams involves either a heavy fermion, say the top quark, or the W boson that couples to an external photon line. In this discussion we shall ignore the charge Higgs loop since they are model dependent and can always be made smaller by making charged Higgs heavier. The strategy is that we like to discuss the consequence of the neutral and charged Higgs mediated CP violation separately and calculate the part of the contribution which is the least model dependent.



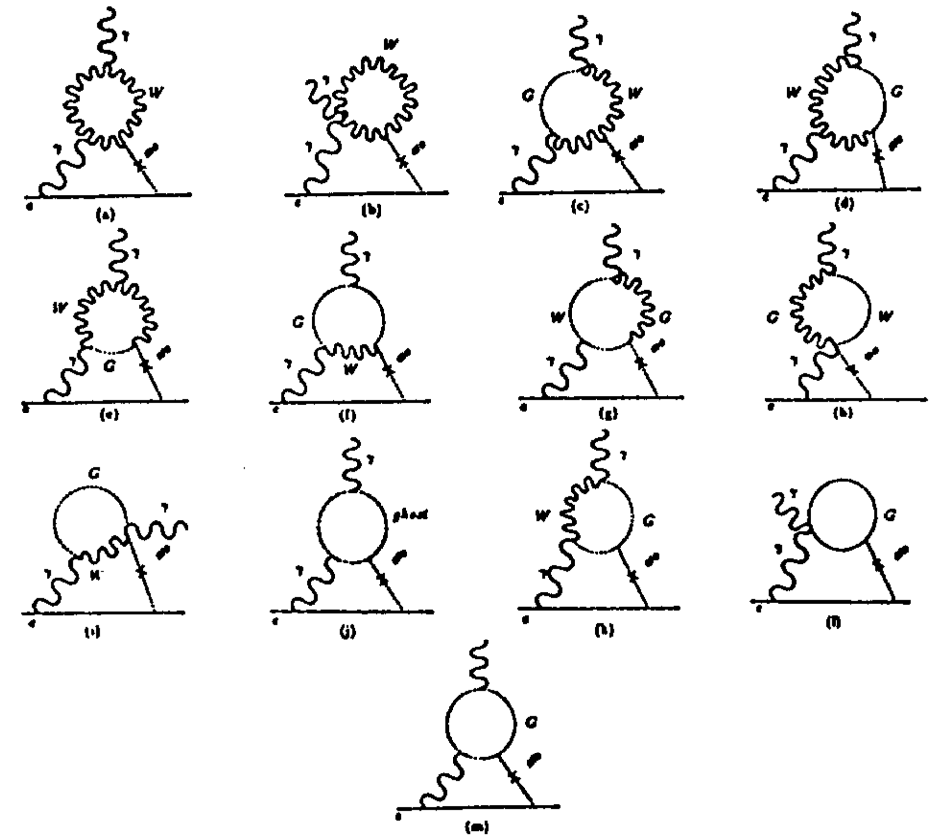

Figure 24: The two Loop diagrams that involve a one loop subdiagram which induces an effective $H\gamma\gamma$ or $HZ\gamma$ vertices

The second class involves a one loop subdiagram that induces an effective electric dipole moment operator for the $W$ boson as in Fig. 25. As we will show in the next section, a general argument of Ref.[81] shows that the two loop diagrams of this type cannot produce a CP violating effective EDM for the $W$ boson in the first loop and hence gives no contribution here. The argument can be generalized to an arbitrary number of loops to show that without using the fermion in the loop the induced $WW\gamma$ vertex can not contribute to the EDM of any fermion in any gauge theory of CP violation. This is because, without fermions, one can find a discrete symmetry, which we shall call V-parity, such that it transforms all the gauge particles like the ordinary parity P but leaves the spinless particles invariant. V-parity forbids any

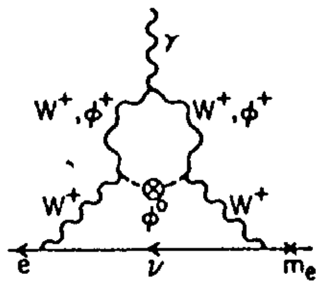


Figure 25: Feynman diagrams for the EDM of the electron with an effective operator for the electric dipole moment of W.

WW$\gamma$ vertex which is P–odd. Therefore the only CP violating WW$\gamma$ vertex that can be induced through bosonic loops has to be P–even and C–odd. To generate the EDM of fermions, the photon field in the WW$\gamma$ vertex has to be in the gauge invariant form, $F^{\mu\nu}$. One can show that, in this case, the WW$\gamma$ vertex is always C even and no EDM of fermions can be induced. As a result, we need the scalar Higgs coupling in the first loop and then the pseudoscalar Higgs coupling to the electron line in the second loop so as to produce the scalar-pseudoscalar mixing which is CP non-conserving.[73]

There are a lot of other diagrams, shown in Fig. 26, that do not contain a one loop subdiagram as in Fig. 23 which induces a bosonic effective vertex. These diagrams were discovered by Leigh, Paban and Xu[74]. Their calculations show that this class of diagrams does not give the dominant contribution.

The dominant contributions are due to the graphs in Fig. 24. Detailed analysis of these diagrams has been done by many authors [73, 74, 75]. We will simply mention the subtlety involved in the calculation. The amplitudes for the effective $H\gamma\gamma$ and $HZ\gamma$ vertices due to the W–loop in the standard model are given in Ref.[77] The result has been confirmed by more than one group. We can easily translate their results into the case of the multi-Higgs doublet models. To do the translation, one notes that, in $R_\xi$ gauge, the diagrams associated with W-loop can be separated into two



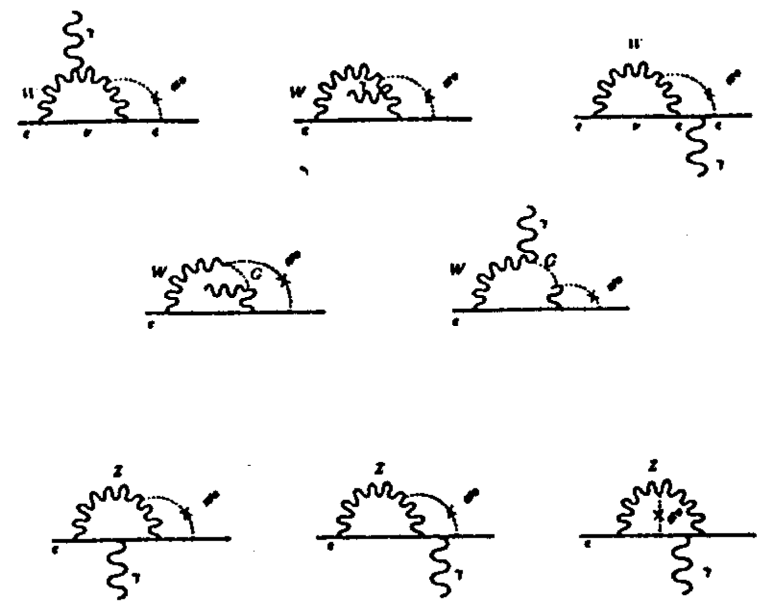

Figure 26: Two loop diagrams that do not contain a one loop subdiagram with effective $H\gamma\gamma$ or $HZ\gamma$ vertex.

gauge invariant sets. The first set involves loops containing the $W$ boson or its ghost while the second set involves specifically the Higgs boson coupling to the unphysical charged Higgs $G^{\pm}$ associated with the W boson. The latter coupling is proportional to the Higgs mass, $M_H^2$, and therefore this set of diagrams forms a gauge invariant subset by themselves. These two sets of diagrams should be translated separately.

For multi-Higgs doublets models, there are more than two CP violating mixings in the neutral Higgs sector[12]. They can be parametrized in terms of Im $Z_i$, Im $Z_{ij}$, and Im $\tilde{Z}_{ij}$ as defined in Eqn.( 27). They are constrained by unitary condition, Eqn.( 29), as derived in Section I.C.. These couplings can be used to translate the set of diagrams with bosonic loops whose coupling constants do not have explicit dependence on the

Higgs mass. But before we get to the explicit two loop results we should step back and see if we can get a reasonable estimate without getting into the details. The idea is to demonstrate how much of the result can be anticipated without the detail calculation just by analyzing the physics carefully.

For the top quark loop, since the Yukawa coupling of the t quark already flips the helicity once in the loop, one needs another mass insertion in the top loop. After that insertion, we will have an infrared singularity in the top quark loop if we set $m_t$ to zero. At the same time since we have already taken the electron propagator to be a massless one in our approximation, even the second loop will also get an infrared divergent in the limit of vanishing $m_t$ Therefore, we expect the $m_t$ dependence of the loop to be $m_t^2(\ln m_t)^2$ for the case of $m_t \ll M_H$. The Yukawa couplings are $m_t/\lambda_2 = (gm_t/2m_W)\cdot(V/\sqrt{2}\lambda_2)$ for the t quark and $m_l/\lambda_1$ for electron, where we assume that the electron couples to $\phi_1$. For the Higgs boson propagator with CP violating mixing, one can write it as $\mathrm{Im}\left\langle\phi_1^0\phi_2^{0(*)}\right\rangle$ where each field can be complex conjugated depending on the helicity of the Yukawa vertices. It is helpful to combine couplings with the propagator in the analysis. If the electron vertex is scalar and top vertex is pseudoscalar, the propagator combined with coupling constants is

$$
\begin{aligned}
& \frac{m_t m_e}{4}\left\langle\frac{\phi_1^0}{\lambda_1}+\frac{\phi_1^{0*}}{\lambda_1^*} \mid \frac{\phi_2^0}{\lambda_2}-\frac{\phi_2^{0*}}{\lambda_2^*}\right\rangle \\
= \; & i\frac{m_t m_e}{4}\left[2\mathrm{Im}\frac{\left\langle\phi_2^0+\phi_1^{0*}\right\rangle}{\lambda_2\lambda_1^*}+2\mathrm{Im}\frac{\langle\phi_2^0\phi_1^0\rangle}{\lambda_1\lambda_2}\right] \\
= \; & i\frac{m_t m_e}{2}\sqrt{2}G_F\frac{\mathrm{Im}Z_{12}^0+\mathrm{Im}\tilde{Z}_{12}^0}{(q^2-M_H^2)}. \qquad (92)
\end{aligned}
$$

Similarly if the electron vertex is pseudoscalar and the top vertex is scalar then we only have to switch between $\phi_1^0$ and $\phi_2^{0*}$ and obtain

$$
i\frac{m_t m_e}{2}\sqrt{2}G_F\frac{\mathrm{Im}Z_{12}^0-\mathrm{Im}\tilde{Z}_{12}^0}{(q^2-M_H^2)}. \qquad (93)
$$

Putting these together, we can estimate the top quark loop contribution to be

$$
\begin{aligned}
\left[\frac{D_e}{e}\right]_t = \; & e^2\left(\frac{1}{16\pi^2}\right)^2\left(\frac{m_t m_e G_F}{\sqrt{2}}\right)\cdot\frac{m_t}{M_H^2} \\
& \left[F\left(\frac{m_t^2}{M_H^2}\right)(\mathrm{Im}Z_{12}^0-\mathrm{Im}\tilde{Z}_{12}^0)+G\left(\frac{m_t^2}{M_H^2}\right)(\mathrm{Im}Z_{12}^0+\mathrm{Im}\tilde{Z}_{12}^0)\right], \qquad (94)
\end{aligned}
$$



The functions $F(z), G(z)$ are of order 1 of $z \sim 1$ and $F(z), G(z) \simeq constant \times (\ln z)^2$ if $z$ is small. Two powers of $M_H$ are present by dimensional arguemnt.

This can be compared with the explicit two loop results [70, 73, 74, 75, 76]. For diagrams with the $H\gamma\gamma$ vertex, it is given by

$$
(D_e/e)^{H\gamma\gamma}_{t-\mathrm{loop}} = -\frac{G_F m_e \alpha}{6\sqrt{2}\pi^3}\sum_n\left[[f(z_{H_n})+g(z_{H_n})]\mathrm{Im}\;Z_{21}^n-[f(z_{H_n})-g(z_{H_n})]\mathrm{Im}\;\tilde{Z}_{21}^n\right]. \qquad (95)
$$

where

$$
\begin{aligned}
f(z) &= \frac{1}{2}z\int_0^1 dx\frac{1-2x(1-x)}{x(1-x)-z}\ln\frac{x(1-x)}{z} \\
g(z) &= \frac{1}{2}z\int_0^1 dx\frac{1}{x(1-x)-z}\ln\frac{x(1-x)}{z} \qquad (96)
\end{aligned}
$$

and $z_{H_n}$ is $m_t^2/M_{H_n}^2$. These functions are such that[70] $f(1) \simeq 1/2, g(1) \simeq 1$; for large $z$, $f(z) \simeq (1/3)\ln z, g(1) \simeq (1/2)\ln z$; for small $z$, $f(z) \simeq g(1) \simeq (z/2)(\ln z)^2$. This clearly confirm our estimate.

For the $HZ\gamma$ vertex, similar estimate applies. However, the Z coupling to the fermion is different from the photon coupling. To start with, the Z boson has both vector and axial vector couplings. However, for fermion loop, Ferry's theorem implies that only the vector coupling of the Z contribute. Ferry's theorem is result of charge conjugation symmetry. The axial vector coupling is odd under charge conjugation. (Reader is reminded that both scalar and pseudoscalar couplings are even under charge conjugation). One may ask why should charge conjugation be a relevant symmetry here since it is well known that the weak interactions does not respect charge conjugation symmetry. The argument here is not too different out arguments in Part I about how one see that CP is broken. For fermion loop, one use only a small subset of the weak interactions. As long as bosons are external fields we may as well pretend that there are two different external particles, one with the vector coupling and the other one with the axial vector coupling to the fermion for symmetry consideration. In that case one can define a charge conjugation symmetry as long as one assigns different $C$ quantum number to the two different Z with vector or axial vector couplings. This also tell us Ferry's theorem will not be applicable at two loop level in which one is allowed to add an Z boson exchange to the fermion loop. In that a situation of vector- axial-vector mixing happen in the propagator not very different from the case of scalar-pseudoscalar mixing for the spin zero case.

As long as the Z coupling to the top quark is only vectorial in the loop, one can conclude that its coupling to the electron has to be also vectorial. Otherwise, we will have an vector-axial-vector mixing in the diagram which is odd under charge conjugation. This will be the only factor which is odd under charge conjugation in the two loop diagram. Since the operator we like to generate is $C$ even, we conclude that this type of mixing can not happen and therefore the Z couplings to both fermions have to be vectorial.

Through these argument we can immediately translate the $H\gamma\gamma$ vertex into the $HZ\gamma$ vertex, we have

$$\begin{aligned}(D_e/e)^{HZ\gamma}_{t-\mathrm{loop}} &= -\frac{(1-4\sin^2\theta_W)(3-8\sin^2\theta_W)}{32\sin^2\theta_W\cos^2\theta_W}\frac{G_F m_e\alpha}{6\sqrt{2}\pi^3}\times\sum_n \\ &\Big\{[\bar{f}(x_{H_n},x_Z) + \bar{g}(x_{H_n},x_Z)]\mathrm{Im}\, Z^n_{21} - [\bar{f}(x_{H_n},x_Z)-\bar{g}(x_{H_n},x_Z)]\mathrm{Im}\,\tilde{Z}^n_{21}\Big\}\,,\end{aligned} \tag{97}$$

with $x_Z = m_t^2/M_Z^2$ and $\bar{f}(x,y) = yf(x)/(y-x) + xf(y)/(x-y)$ and similarly for $\bar{g}$. Note that numerically since $4\sin^2\theta_W$ is about one, the Z contribution is much less significant than the photon contribution.

For the $H\gamma\gamma$ vertex with the W boson loop, one can do similar estimate. However one should know where the Higgs couplings to the W boson are derived. The coupling arise from the gauge covariant kinetic term of the Higgs fields. For a multiHiggs model the Feynman rule for $WW\phi_i$ vertex is $ig_{\mu\nu}g^2\sum_i \mathrm{Re}(((\phi_i^0)^*)\,\phi_i^0)$. Therefore the Yukawa coupling to the electron has to be purely pseudoscalar in order to get CP violation. This together with the Higgs propagator and the Yukawa coupling to electron gives

$$\begin{aligned}& ig^2\frac{-im_e}{2}\left\langle \mathrm{Re}((\lambda_i^*)\,\phi_i^0)\mid \frac{\phi_1^0}{\lambda_1}-\frac{\phi_1^{0*}}{\lambda_1^*}\right\rangle \\ =\;& \frac{g^2}{2}m_e i\left[|\lambda_1|^2\mathrm{Im}A_1 + \sum_{i=2}^N|\lambda_i|^2(\mathrm{Im}\tilde{A}_{i1}-\mathrm{Im}A_{i1})\right] \\ =\;& -ig^2 m_e\sum_{i=2}^N|\lambda_i|^2\mathrm{Im}A_{i1},\end{aligned} \tag{98}$$



where in the last equality we have used the unitary gauge condition in Eqn.( 29) in Section I.C.. Therefore we expect the answer to be proportional to $\mathrm{Im}Z^0_{k1}$ and independent of $\mathrm{Im}\tilde{Z}^0_{k1}$.

Two loop calculation for the $H\gamma\gamma$ case gives the electron EDM

$$(D_e/e)^{H\gamma\gamma}_{W-\mathrm{loop}} = \frac{G_F m_e\alpha}{8\sqrt{2}\pi^3}\sum_n \eta_n\left[3f(z_{H_n}) + 5g(z_{H_n}) + \tfrac{3}{4}g(z_{H_n}) + \tfrac{3}{4}h(z_{H_n})\right]\,, \tag{99}$$

where we follow the notations of Refs.[70, 73] with $\eta_n = \Lambda^{-2}\sum_{k=2}^N|\lambda_k|^2\mathrm{Im}\,Z^n_{k1}$, $\Lambda^{-2} = \sum_k|\lambda_k|^2$ and $z_{H_n} = M_W^2/M_{H_n}^2$. The function $h(z)$ is defined to be

$$h(z) = \frac{z}{2}\int_0^1\frac{dx}{z-x(1-x)}\Big[1+\frac{z}{z-x(1-x)}\log\frac{x(1-x)}{z}\Big]. \tag{100}$$

The first two terms in Eqn.( 99) arise from the terms in the $H\gamma\gamma$ vertex which are linear in the external photon momentum while the next two terms arise from the terms which are independent of the momentum. In the latter case one has to expand the propagators in the second loop to get the external momentum factor.

For the $HZ\gamma$ case, one has

$$\begin{aligned}(D_e/e)^{HZ\gamma}_{W-\mathrm{loop}} &= \frac{1-4\sin^2\theta_W}{4\sin^2\theta_W}\frac{G_F m_e\alpha}{8\sqrt{2}\pi^3}\sum_n\eta_n\Big[\tfrac{1}{2}(5-\tan^2\theta_W)\bar{f}(z_{H_n},z_Z) \\ &+ \tfrac{1}{2}(7-3\tan^2\theta_W)\bar{g}(z_{H_n},z_Z) + \tfrac{3}{4}g(z_{H_n}) + \tfrac{9}{4}h(z_{H_n})\Big].\end{aligned} \tag{101}$$

where $z_Z = M_W^2/M_Z^2$. Note that only the vector part of the $Z$–$\bar{e}$–$e$ vertex contributes to the CP violating EDM operator and thus produces the suppression factor of $(1-4\sin^2\theta_W)$ in Eqn.( 101). If one assumes, for simplicity, that the lightest Higgs boson $H_0$ dominates and the other heavier Higgs boson can be neglected, then the numerical result due to the contributions from Eqns.( 99, 101) are shown in Fig. 27 with $\eta_0 = \frac{1}{2}$.

The W–loop contribution of $HZ\gamma$ is about 10% of that of $H\gamma\gamma$ and they have the same sign.

It is much more ambiguous to translate the subset of diagrams involving the unphysical Higgs $G^\pm$, one needs the coupling of the physical Higgs boson to the unphysical Higgs pair $G^+G^-$. This can be shown to be

$$\mathcal{L} = -\Lambda^{-2}\sum_{i,j}\lambda_i M^2_{ij}G^+G^-\phi_j^0 + \cdots\,, \tag{102}$$

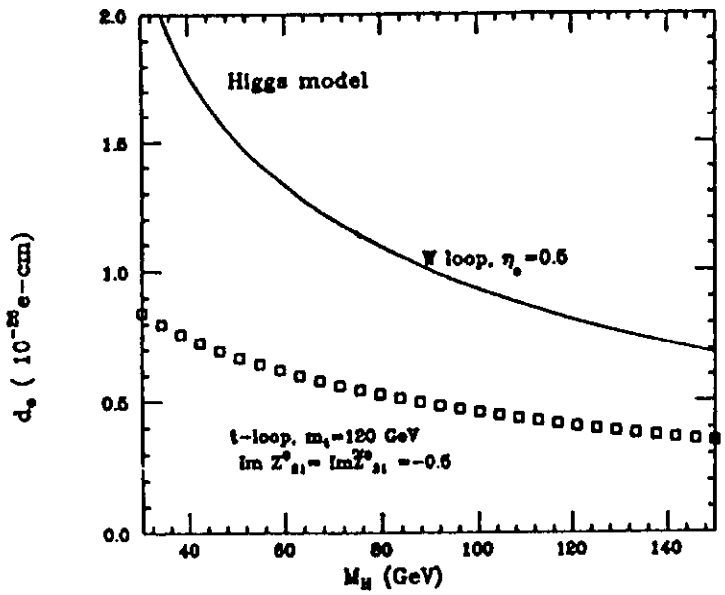


Figure 27: Numerical estimate of the $d_e/e$ via the $W$-loop when $\eta_n = \frac{1}{2}$. The data points show the contribution due to the top quark loop for the case $\mathrm{Im}Z^0_{21} = \mathrm{Im}\tilde{Z}^0_{21} = -\frac{1}{2}$ and $m_t = 120$ GeV.

where $M^2_{ij}$ is the $N \times N$ submatrix of the neutral Higgs mass matrix associated with $\phi_i\phi_j$. Using this coupling, we derive

$$(D_e/e)^{H\gamma\gamma}_{G\text{-loop}} = \frac{G_F m_e \alpha}{16\sqrt{2}\pi^3}\sum_n \frac{\eta_n}{z_{H_n}}\left[f(z_{H_n}) - g(z_{H_n})\right] , \tag{103}$$

and

$$(D_e/e)^{HZ\gamma}_{G\text{-loop}} = \frac{1-4\sin^2\theta_W}{8\sin^2\theta_W}\frac{G_F m_e \alpha}{16\sqrt{2}\pi^3}\sum_n \frac{\eta_n}{z_{H_n}}(1-\tan^2\theta_W)\left[\tilde{f}(z_{H_n}, z_Z) - \tilde{g}(z_{H_n}, z_Z)\right] . \tag{104}$$

The amplitude for each Higgs boson increases logarithmically with the Higgs boson mass. In this case, the lightest Higgs contribution may no longer be the most important one. This makes reliable estimate of this type of contribution difficult. However, the coefficients are small enough that these contributions may not be so significant as compared to the W-loop contribution discussed earlier except for the case of very



heavy Higgs boson.

Numerically the contribution from the top quark is generally smaller than that from the W boson. We demonstrate this point in Fig. 27 by choosing typical values of the CP violating parameters $\mathrm{Im}Z^0_{21} = \mathrm{Im}\tilde{Z}^0_{21} = -\frac{1}{2}$ with $m_t = 120$ GeV. It is worth mentioning that the t-quark loop contribution involves a linearly independent combination of CP violating parameters, $\mathrm{Im}Z^0_{k1}$, as compared to the W-loop or G-loop contributions.

To summarize, the W-loop contributions, ignoring the G-loop subset that involves the Higgs boson mass in the vertex, may provide the majority of the contribution. The G-loop subset may become important when the Higgs boson mass is very heavy, in that case, the Higgs sector may be strongly interacting and a reliable estimate is very difficult.

*(2) chromo-electric dipole moment of quarks* The mechanism discovered by Barr and Zee can also leads to a large chromo-electric dipole moment of the light quarks and thus gives rise to a neutron electric dipole moment which is almost two orders of magnitude larger than the usual one loop mechanism. Either the neutral Higgs boson has to be very heavy or the complex CP violation phase has to be very small.

If the CP violation arises from a Higgs sector that obeys natural flavor conservation then the Higgs bosons can be relatively light as mentioned before[12, 6, 8, 9, 11, 10]. If one uses the charged Higgs sector to explian $\epsilon$, the we can use it to constrain CP violating parameter. In turn, we can use the result to predict $D_N$. For a charged Higgs boson mass about $10GeV$, the one loop contribution to $D_N$ is estimated to be $-9\times10^{-26}e-cm$[15] which is not too far from the experimental upper bound of $8\times10^{-26}e-cm$[50, 52]. Such a light charged Higgs boson is no longer realistic. For a $100GeV$ charged Higgs boson, the one loop estimate of $D_N$ is about $10^{-27}e-cm$. The neutral Higgs boson can also provide interesting contribution to $D_N$. The naive estimate of its one loop contribution gives small result because it is suppressed by three powers of light quark mass as mentioned in Section I.C.. A more carefull analysis indicates that if one considers the neutral Higgs boson coupling to the nucleons instead of quarks the effect can be much larger. A recent estimate[15] gives $2\times10^{-26}(100GeV/M_{H^0})^2e-cm$.

The Barr and Zee's two loop mechanism provides another way to avoid the light quark mass suppression effect. It turns out the leading contribution to the two loop mechanism are the chromo-electric dipole moments (CEDMs) of the light quarks. This is anticipated because when the two photons in Fig. 22 are replaced by two gluons one gets an enhancement factor of $\alpha_s/\alpha$. However, note that the loop momenta in the two loop diagram are both of high mass scale, $M$ (which is $m_t$ for the most part). Therefore the CEDM is induced at the scale $M$. When the QCD renormalization group correction is taken into account, the scaling of the CEDM operators from the high mass scale ($M$) down to the hadronic scale ($\mu$) give rise to a suppression effect of about one percent. This suppressive effect is the same as the one we had discussed in Section II.A. for the operator $O_q$.

The calculation of the gluonic diagrams is very similar to the photonic ones discussed before. For gluonic case, we will have to insert soem color factors. It gives rise to the CEDMs, $D^c_{u,d}$, for the up and down quarks,[76]

$$d^c_u = g_s f_u = 2m_u(\frac{g_s}{4\pi})^3\sqrt{2}G_F\frac{\mathrm{Im}Z_0 - \mathrm{Im}\tilde{Z}_0}{\tan^2\beta}(f(z)+g(z)) \qquad (105)$$

$$d^c_d = g_s f_d = 2m_d(\frac{g_s}{4\pi})^3\sqrt{2}G_F\left[\mathrm{Im}Z_0(f(z)+g(z)) - \mathrm{Im}\tilde{Z}_0(f(z)-g(z))\right] \qquad (106)$$

where $f(z)$ and $g(z)$ were defined in Eqn.( 96) and $z$ is $m_t^2/M_{H^0}^2$. As expected, this two loop mechanism requires only one power of light quark mass suppression. The ratio of the vacuum expectation values of the Higgs bosons, $v_2/v_1$, is defined to be $\tan(\beta)$.

The CEDM operators of the light quarks are induced at the heavy t quark or Higgs boson mass scale when these particles are integrated out in the effective theory. QCD renormalization effect brings us an extra factor

$$\left(\frac{g_s(M)}{g_s(\mu)}\right)^{\frac{14}{23}} \qquad (107)$$

where we have assumed five flavors between the two scales $M$ and $\mu$. The strong coupling constant $g_s$ in Eqns.( 105) and ( 106) has to be replaced by the running coupling $g_s(\mu)$. To estimate the resulting neutron electric dipole moment, we employ the valence quark model which gives

$$D_N = \frac{1}{3}e(\frac{4}{3}f_d + \frac{2}{3}f_u). \qquad (108)$$



Therefore the two loop contributions to the neutron electric dipole moment due to the CP violating neutral Higgs boson exchange is

$$\begin{aligned} D_N &= \frac{4}{9}\left(\frac{e}{4\pi}\right)\left(\frac{g_s(\mu)}{4\pi}\right)^2\left(\frac{g_s(M)}{g_s(\mu)}\right)^{\frac{14}{23}}\sqrt{2}G_F[(\mathrm{Im}Z_0 - \mathrm{Im}\tilde{Z}_0)(2m_d + \frac{m_u}{\tan^2\beta})f(z) \\ &+ (\mathrm{Im}Z_0(2m_d + \frac{m_u}{\tan^2\beta}) + \mathrm{Im}\tilde{Z}_0(2m_d - \frac{m_u}{\tan^2\beta}))g(z)] \end{aligned} \qquad (109)$$

To estimate its magnitude, assume the neutral Higgs boson to have about the same mass as the t quark and assume the CP violating quantities $\mathrm{Im}Z_0$ and $\mathrm{Im}\tilde{Z}_0$ to be of order unity. Take $M \sim 100GeV$, $g_s(M)/4\pi$ is about 0.1, and following Weinberg[55], use $g_s(\mu)/4\pi = 1/\sqrt{6}$. This gives $D_N$ to be about $7.8 \times 10^{-25}(m_d/10MeV)e-cm$. Therefore even if we conservatively used the current quark masses the contribution is almost two orders of magnitude larger than the one loop estimate quoted earlier under the same assumptions about the neutral Higgs boson mass and CP violating parameters. If the constituent quarks masses are used instead, one can boost up at least one order of magnitude of the above estimation of $D_N$. Also, this is already about two orders of magnitude larger than the neutral Higgs boson contribution through the two loop induced chromo-electric dipole moment of the gluon under the same assumptions[15]. Barr and Zee[70] also calculated the electric dipole moments of the light quarks in the photonic two loop mechanism. The gluonic two loop contribution discussed here is actually larger than the photonic one. This gluonic two loop mechanism therefore provides the most stringent constraints on the CP violating parameters and/or the neutral Higgs boson mass. Of course, one should also keep in mind that the estimate of $D_N$ is necessarily more uncertain than the photonic one due to the low energy hadronic physics. Using this mechanism, one can already rule out some of the models of CP violation which use the neutral Higgs bosons as the main source of CP violation[11].

## C. Electric Dipole Moment of W boson

Here we like to discuss another recent development about CP vioaltion, the electric dipole moment $D_W$ of the $W$ gauge boson, $D_W$.

Immediately after the discovery of CP violation, it was suggested by Salzman and Salzman[78] and others[79, 80] that the observed CP violation in the neutral kaon system might result from an intrinsic electric dipole moment of $W$. The order-of-magnitude of the CP violation parameter $\epsilon$ in the kaon decay is very close to $\alpha/\pi$ and hence this suggested a CP violating electromagnetic effect on the weak interaction amplitudes. It was also realized that a finite $D_W$ could induce an electric dipole moment for the neutron.

There are two levels one can pose questions about $D_W$. The first one is, given a model of CP violation, how large is the induced $D_W$. This question has been investigated to one loop level in ref.[81, 82] In particular, it was found that there is no one loop contribution in models in which there is no right-handed vector current. Models in which CP violation is mediated by Higgs bosons are examples of this type. In models in which CP violation is mediated by neutral Higgs bosons, a two loop analysis of $D_W$ has been presented in ref.[83]. Two loop contributions to $D_W$ for models in which CP violation is mediated by charged Higgs bosons has also been done[84]. In this section we like to review these calculations.

At the second level, one likes to know given a nonzero electric dipole moment for $W$ what kind of physical measurement will constrain its value. Marciano and Queijeiro[85] updated the orignal analysis of Salzman and Salzman[76] and they found that measurements of the neutron electric dipole moment of the order $10^{-25}e-cm$ could be used to place a very stringent upper bound on $D_W$ for its absolute magnitude

$$|D_W| \leq 10^{-20}\ \text{e--cm}. \tag{110}$$

They have assumed a resonable form factor to tame the divergence. Unless this form factor suppression is much stronger, otherwise the restriction eliminates the possibility of using $D_W$ to explain $\epsilon$. However, the effect of $D_W$ of the order of $10^{-20}e-cm$ may still be accessible[86] in certain processes, for instance in the scattering $\gamma e^{\pm} \rightarrow W^{\pm}\nu$, for future experiments. Also, careful studies of the polar and azimuthal distributions of leptons and antileptons produced in $W$ decays[87] may further provide useful constraints on the size of $D_W$. With the increasing production luminosity of $W$ pairs in laboratories, it becomes of current interest to estimate the size of $D_W$ in various CP-violating gauge models.

As it is well known that if $P$- and $T$-symmetries are violated, elementary particles with spin degrees of freedom may have electric dipole moments. The most general



form of the $W$ boson coupled to a photon has seven terms[87] among them two of which violate $P$- and $T$- and hence CP-symmetries,

$$ikW_\mu^\dagger W_\nu \tilde{F}^{\mu\nu} + i(\lambda/M_W^2)W_{\sigma\mu}^\dagger W_\nu^\mu \tilde{F}^{\nu\sigma}. \tag{111}$$

Here $W_\mu$ is the $W^-$ gauge potential, $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + \cdots$, and the dual of the photon field strength is $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha)$. In the momentum space, these terms can be expressed as

$$f_1(q)\epsilon^{\mu\nu\alpha\beta}(p-p')_\beta + (f_2(q)/M_W^2)\epsilon^{\mu\nu\rho\beta}(p-p')_\beta(p+p')^\alpha(p+p')_\rho. \tag{112}$$

Here $p$ and $p'$ are the incoming and the outgoing momenta of the W boson. The form factors $f_1 = \lambda - k$ and $f_2 = \frac{1}{2}\lambda$ are functions of $(p-p')^2$ (the square of the momentum transfer). The electric dipole moment $D_W$ can be expressed[87] in terms of these form factors in the limit $(p-p')^2 \rightarrow 0$ in the unit of $e/2M_W = 1.2 \times 10^{-16}$ e-cm,

$$D_W = (f_1 - 4f_2)(e/2M_W)\ . \tag{113}$$

In gauge theories, a CP-violating but $SU(2)_L$ invariant term $\theta W^{\mu\nu}\tilde{W}_{\mu\nu}$ can be added to the Lagrangian. However, this term can be rewritten as a total divergence and thus will not contribute to $D_W$ perturbatively. Nonperturbative effects due to such a term is suppressed at least by a factor[88] $\exp(-8\pi^2/g^2)$, where $g$ is the weak coupling constant, and hence extremely small. In what follows we will ignore this contribution. Also, note that the first term in Eq. (2) has a dimensionality of four. However, this term is not invariant under $SU(2)_L \times U(1)$ and, therefore, can only be generated through higher dimensional gauge invariant terms of the form $\phi^n W^\dagger W \tilde{A}$. Here $\phi^n$ represents, generically, an interaction of $n$ neutral Higgs fields. Since the other term already has a dimensionality of six, the CP-violating electromagnetic form factor is thus induced by operators with a dimensionality greater than four. As a result, $D_W$ is calculable even in models with "*hard*" CP-violation. Note that the second term can be $SU(2)_L \times U(1)_Y$ invariant if all three fields arise from $SU(2)_L$.

It is interesting to note that Marciano and Queijeiro[85] used only the dimension 4 term in Eq.( 111) as effective interaction to calculate the neutron electric dipole moment induced by it. To regulate the divergent integral, they introduced a form factor,

$$k = \lambda_W \frac{(\Lambda^2 - M_W^2)^2}{(k^2 - k\cdot q - \Lambda^2)(k^2 + k\cdot q - \Lambda^2)} \tag{114}$$

where $q$, $k$ are momenta the photon and the $W$ respectively, to cut off the integral. They found that the experimental constraint on $D_n$ implies that the coupling k has to be $\leq 10^{-3}$. There are two interesting questions one can ask about this result. First of all one like to know how model dependence is the form factor they used. This can be done by analyzing the form factors of some of the models that we will calculate later. This work is still in progress. Secondly since the dimension six operator can give as important contribution to $D_N$ as the dimension four one, one like to ask the same question for this operator as well.

An obvious distinction between the CP-conserving and the CP-violating eletromagnetic form factors of $W$ is that the CP-violating terms are directly proportional to the Levi-Civita tensor. Such a tensor occurs naturally in the spinor trace of the Dirac matrices, $\epsilon^{\mu\nu\alpha\beta} = \frac{1}{4}\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\gamma_5)$. Thus the fermion loop is required to give $D_W$. This fact can be understood in a differnet way. A perturbative renormalizable theory that contains only gauge bosons and Higgs bosons (without fermions) is always invariant under the symmetry $\tilde{P}$: $x^\mu \to x_\mu$, $W^\mu \to W_\mu$, $\phi \to \phi$. Consequently, the lowest order that may potentially contribute to $D_W$ must contain fermion loops.

To study the size of $D_W$ quantitatively, we consider the following general $W$-fermion interaction

$$\mathcal{L}^{cc} = -\frac{g}{\sqrt{2}} W_\mu^+ \sum_{i,j} \bar{f}_i \gamma^\mu (V_{ij} L + U_{ij} R) f'_j + \sum_i (m_i \bar{f}_{Li} f_{Ri} + m'_i \bar{f}'_{Li} f'_{Ri}). \qquad (115)$$

Here $L, R = \frac{1}{2}(1 \mp \gamma_5)$, $i$ and $j$ are generation indices, $f$ and $f'$ represent fermion fields with charges different by one unit. The phases in the mixing matrices $V$ and $U$ are the sources of the CP violation. CP violation requires only one $f$ and one $f'$. We put in more than one just for generality. The CP violation mechanism is the same as the left-right model one discussed is Section I.D.. Clearly, Cp vioaltion requires that our answer be proportional to $m_f m'_f VU$ for the simplest one generation case. The one-loop contributions to $D_W$ are depicted in Fig. 28. Evaluating these graphs, we find

$$D_W = \frac{g^2 C}{8\pi^2} \sum_{a,i,j} \frac{e}{2M_W} \frac{m_i m'_j}{M_W^2} Im(V_{ij} U^*_{ij}) (Q_i \mathcal{I}(\frac{m_i^2}{M_W^2}, \frac{m'^2_j}{M_W^2}) + Q'_j \mathcal{I}(\frac{m'^2_j}{M_W^2}, \frac{m_i^2}{M_W^2}), \qquad (116)$$

where the function is

$$\mathcal{I}(x,y) = \int_0^1 d\alpha \frac{\alpha}{\alpha(\alpha-1) + \alpha x + (1-\alpha) y - i\epsilon} \qquad (117)$$



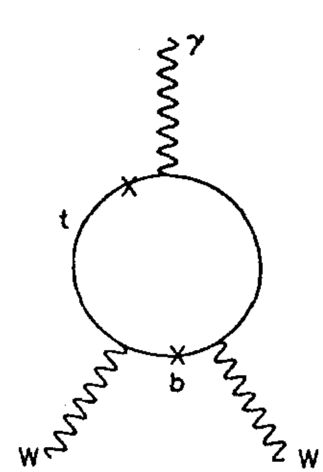


Figure 28: One–loop Feynman graphs for calculating $D_W$ due to the left–handed and the right handed currents.

Here $Q_i$ and $Q'_j$ are the charges for the fermions $f$ and $f'$ respectively. The color factor $C$ is 1 or 3 for the lepton or for the quark. The fermion masses $m_i$ and $m'_j$ occur explicitly in the factor $\frac{m_i m'_j}{M_W^2}$ due to the helicity argument. Typical value of the function $\mathcal{I}$ at the electroweak scale is about unity, *e.g.* $\mathcal{I}(1,1) = \sqrt{3}\pi/9 \simeq 0.6$.

Note that this one loop contribution gives rise to only the dimension four operator in Eqn 111. It seems to be the general case that only the model with some kind of right-handed current can contribute to $D_W$ at the one loop level and at the one loop level the only operator that is induced is the dimension four one. The reason for this is not very clear yet.

In the following we shall go through different models of CP violation and investigate their contribution to $D_W$ (1). In the KM model,[4] the electric dipole moment vanishes at one–loop level because there is no right-handed current ($U_{ij} = 0$). A two loop estimate was given before in Eqn.( 17) of Section I.B.. In fact, one can argue that even at the two loop level the contribution is probably zero. The argument goes as follows. There are only four fermion lines in the two loop diagram. In the unitary gauge, all the interactions are left-handed therefore the quark masses must appear quadratically. In the KM model, the CP violation disappears when any two of the up– or the down–type quarks are degenerate in mass. Therefore we expect any CP violating effect to carry a factor of $\Pi_{i<j}(m^2_{d_i} - m^2_{d_j})(m^2_{u_i} - m^2_{u_j})$. As a result, there

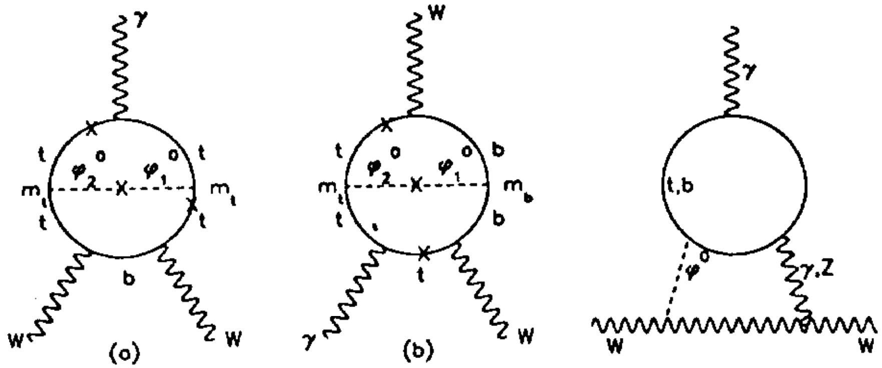

Figure 29: A sample of two-loop Feynman graphs for calculating $D_W$ in the neutral Higgs model of CP violation.

are a total of 6 powers of quadratic mass differences. The GIM effect at each fermion line in the loop yields one factor of quadratic mass differences. Therefore, from a two loop diagram, it can only produce four powers of quadratic mass differences which is smaller than the 6 powers as needed for CP violation. The arguemnt is of course very qualitative. In fact it contradict with a recent claim by Hoogeveen[38] who calculated three loop contributions to the electric dipole moment of the electron. His diagrams contain a two loop subdiagram which is supposed to contribute to $D_W$.

(2). In the Weinberg-Higgs model[6], $U_{ij} = 0$. Consequently, $D_W = 0$ at one-loop level. However, a nonzero $D_W$ can arise through two-loop graphs. The dominant diagrams for neutral Higgs models of CP violation are shown in Fig. 29 Some of the neutral Higgs contributions (Fig. 29(a)) have been recently studied by He and McKellar.[83] The complete two loop amplitude still requires more detailed calculations. Also, they have only calculated the contribution of the diagrams to the dimension four operator in Eqn.( 111). In general, the dimension 6 operator can contribute as large as the dimension four one in this case.

For the charged Higgs models the leading diagrams are shown in Fig. 30. Note that if the charged Higgs mass is as large as the neutral Higgs mass and they have similar CP violating phases then the neutral Higgs contribution will dominate because the charged Higgs diagrams are suppressed by extra powers of lighter quark mass ($m_b$).



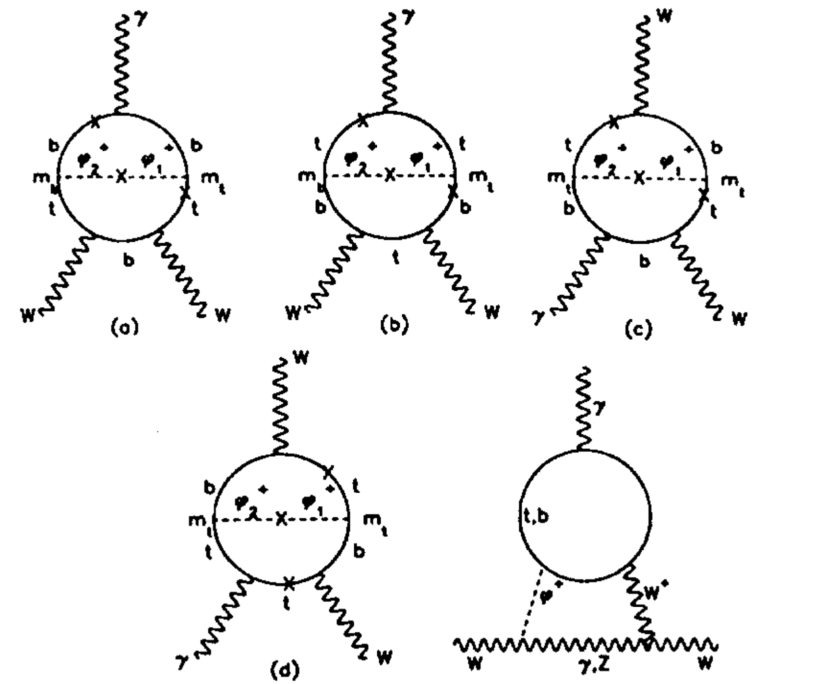

Figure 30: A sample of two-loop Feynman graphs for calculating $D_W$ in the charged Higgs model of CP violation.

However, in many models[6, 10] the charged Higgs can not be too heavy because it has to be used to explain $\epsilon$ while there is no similar situation for neutral Higgs. Therefore it is still very interesting to investiagte charged Higgs contribution[84].

Here we shall only estimate the size of $D_W$ for neutral Higgs models to be of the order

$$D_W \sim \sin\delta_H \Big(\frac{g^2}{8\pi^2}\Big)^2 \Big(\frac{m_f}{M_{H^0}}\Big)^2 \Big(\frac{e}{2M_W}\Big). \tag{118}$$

The CP violating phase $\delta_H$ characterizes the complex mixing in the Higgs sector. It could be of the order of unity. The contribution is large if the the mass $m_{H^0}$ of the neutral Higgs is small. With three generations of fermions, it gives

$$D_W(Higgs\ Model) \le 10^{-20}\Big(\frac{m_t}{100\ GeV}\Big)^2\Big(\frac{10\ GeV}{m_H}\Big)^2 e - cm. \tag{119}$$

For $m_t \sim 100\ GeV$ and $m_{H^0} \sim 10\ GeV$, this yields $D_W \le 10^{-20} e - cm$.

(3). In left-right models[16, 18] the leading contribution should be due to the CP violating phase associate with the left-right mixing. The electric dipole moment $D_W$ arises even for the case of only one generation. We can consider the dominant contribution from the top and the bottom quark generation (assuming $g_L = g_R$).

$$\mathrm{Im}(V_{tb}U^*_{tb}) \simeq \xi \sin\delta_{LR}, \tag{120}$$

where $\xi$ is the left-right mixing which is bound[45] by $\xi \leq 5\times 10^{-3}$. We find the dominant contribution is of the order

$$\begin{aligned} D_W(LRModel) &= \xi \sin\delta_{LR}\frac{g^2}{8\pi^2}\frac{m_b m_t}{M_W^2}\Big(\frac{e}{2M_W}\Big) \\ &\times\Big[2\mathcal{I}\Big(\frac{m_t^2}{M_W^2},\frac{m_b^2}{M_W^2}\Big)-\mathcal{I}\Big(\frac{m_b^2}{M_W^2},\frac{m_t^2}{M_W^2}\Big)\Big] \leq 10^{-22}\mathrm{e}-\mathrm{cm}. \end{aligned} \tag{121}$$

(4). In supersymmetric (SUSY) models[27] the internal fermions in Fig. 28 can be supersymmetric particles —the charginos and the neutralinos. To show the basic mechanism of the CP violation, we consider only the case when the neutralino is the photino $\tilde{\gamma}$ and the chargino is the wino $\omega$. In general, the photino will mix with the neutral higgsino, the zino and the neutrinos; and the wino will mix with the charged higgsino. We avoid this extra complication in the simplified scenario in order the illustrate the physics involved. One can easily extend our approach to the general case. In terms of the independent Weyl's fields $\omega_L^+$, $\omega_L^-$ and $\tilde{\gamma}_L$, the relevant Lagrangian is

$$\mathcal{L} = -eW_\mu^+(\bar{\omega}_L^+\gamma^\mu\tilde{\gamma}_L + \bar{\tilde{\gamma}}_L\gamma^\mu\omega_L^-) - m_{\tilde{\gamma}}\bar{\tilde{\gamma}}_L^c\tilde{\gamma}_L - m_\omega e^{i\delta_S}\bar{\omega}_L^+\omega_L^{-c} + \cdots + h.\ c. \tag{122}$$

The mass terms of the wino and the photino break supersymmetry softly. A phase $\delta_S$ in the mass term is usually allowed and it cannot be totally absorbed by redefining the fields. Consequently, this causes CP non-conservation. To recover the usual form of the mass expression, we define the Dirac field $\omega^+ = \omega_L^+ + e^{i\delta_S}(\omega_L^-)^c$ and the Majorana field $\tilde{\gamma} = \tilde{\gamma}_L + \tilde{\gamma}_L^c$. The above Lagrangian becomes

$$\mathcal{L} = -eW_\mu^+\bar{\omega}^+\gamma^\mu(L - e^{i\delta_S}R)\tilde{\gamma} - \frac{1}{2}m_{\tilde{\gamma}}\bar{\tilde{\gamma}}\tilde{\gamma} - m_\omega\bar{\omega}^+\omega^+ + \cdots . \tag{123}$$

Both the left-handed and the right-handed currents appear with a relative phase $\delta_S$. We can obtain $D_W$ from Eqns.( 115– 117),

$$D_W = \frac{e^2}{4\pi^2}\Big(\frac{e}{2M_W}\Big)\frac{m_{\tilde{\gamma}}m_\omega}{M_W^2}\sin\delta_S\,\mathcal{I}\Big(\frac{m_\omega^2}{M_W^2},\frac{m_{\tilde{\gamma}}^2}{M_W^2}\Big). \tag{124}$$



Usually there could be additional factors due to the the mixings among charginos and neutralinos. At present, no direct phenomenological constraint on these mixings is available. Also, the CP-violating phase $\delta_S$, allowed by the soft supersymmetry breaking Lagrangian, could be naturally of the order of unity. The only natural suppression on $D_W$ in this class of models is thus the loop factor $e^2/4\pi^2$. As a result, we expect that in supersymmetric models $D_W$ could be as large as the present limit given by Eqn. 110

$$D_W(SUSY\ Model) \leq 10^{-20}\mathrm{e}{-}\mathrm{cm}. \tag{125}$$

(5). In mirror models[29] the presence of mirror quarks and mirror leptons introduces right-handed currents with $W$. The mixing between a quark and its mirror image, $\xi_q$, is strongly constrained by the absence of flavor changing neutral current, where one finds[89]

$$\xi_q \leq 10^{-3} - 10^{-4}. \tag{126}$$

The relevant Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} &= -\frac{g}{\sqrt{2}}W_\mu\Big(\bar{u_L}\gamma^\mu d_L + \bar{U}_R\gamma^\mu D_R\Big) \\ &+M_u\bar{u}_R U_L + M_d\bar{d}_R D_L + M_2(\bar{u}_L U_R + \bar{d}_L D_R) \\ &+m_u\bar{u}_L u_R + m_d\bar{d}_L d_R + m_U\bar{U}_L U_R + m_D\bar{D}_L D_R + \cdots . \end{aligned} \tag{127}$$

Note that $M_i$ are $SU(2)_L$ invariant masses and $m_i$ are $SU(2)_L$ broken masses. It is reasonable to assume that $m_u, m_d$ are the smallest massive parameters. Also, the constraint on mirror mixing, Eqn.( 126) requires $m_U, m_D \gg M_{u,d,2}$. It is possible to define the fields so that only $m_u, m_d$ are complex, *i.e.* CP violating parameters. Therefore the CP violating effect should be proportional to either $m_u$ or $m_d$. Now, it is easy to draw diagrams that will contribute to $D_W$. Some typical ones are shown in Fig. 31. Each $M_i$ insertion corresponds a factor of $\xi$ mixing. It then follows from Eqns.( 115– 117) that contributions to $D_W$ from virtual quark-mirror-quark exchange is typically of the order

$$D_W \sim \frac{g^2}{8\pi^2}\xi_q^2\sin\delta_M\Big(\frac{e}{2M_W}\Big)\frac{m_q}{m_Q} \leq 10^{-25}e - cm. \tag{128}$$

The phase $\delta_M$ characterizes the complex mixing among the quark and its mirror. A similar size of contribution can also be generated from lepton–lepton mixings. It

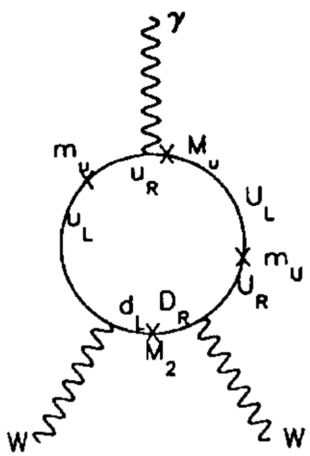


Figure 31: Typical graph for $D_W$ in the mirror models with the relevant mass insertions illustrated.

should also be pointed out that the constraint on $\xi_t$ can be evaded if (1) there is a fourth generation and (2) its mixing with the rest of the generations are negligible. In that case, we find $D_W$ can be as large as of the order $10^{-20}e-cm$.

## Conclusion

We have reviewed the basic mechanisms of CP violation in gauge theory. We have also used the recent development in CP violation as example to illustrate how these mechanisms work in producing CP violating phenomenology. There are a lot of very exciting CP phenomenology we have not been able to cover due to limiting in space and time. Examples are CP violation in the B system or CP violation in $K_{\mu 3}$ decay, hyperon decay. However armed with the mechanisms and the examples illustarted in this paper, the reader may be able to do some exploring of his own.

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